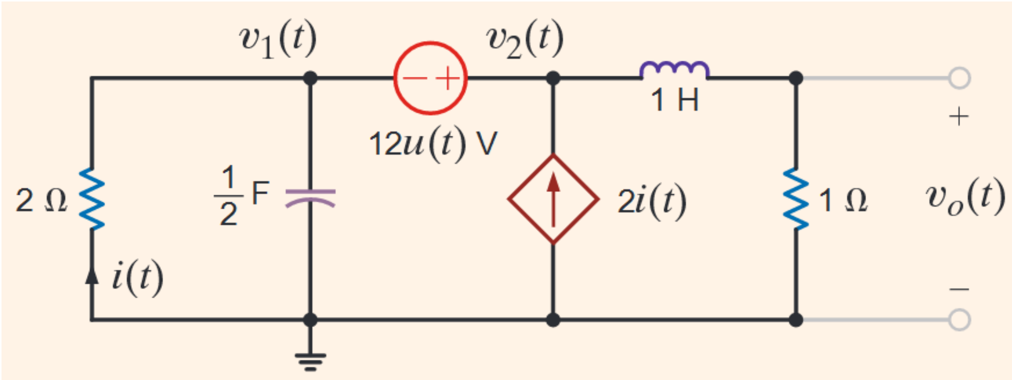
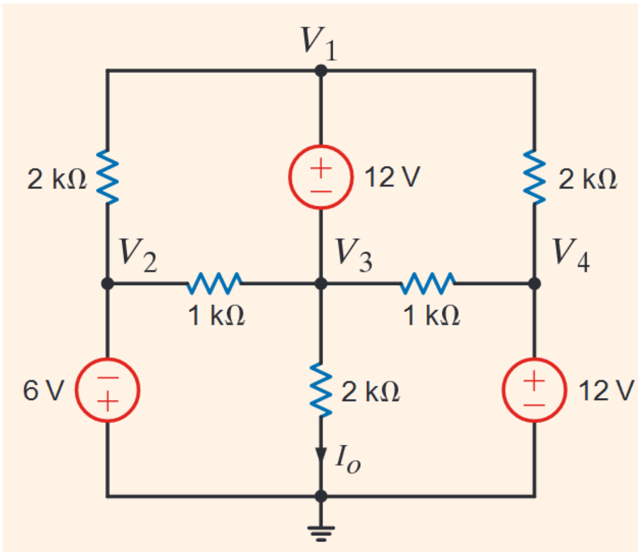
IZMIR KATIP CELEBI UNIVERSITY
DEPARTMENT OF BIOMEDICAL ENGINEERING
FINAL EXAM OF BME203 CIRCUIT THEORY

January 13, 2017

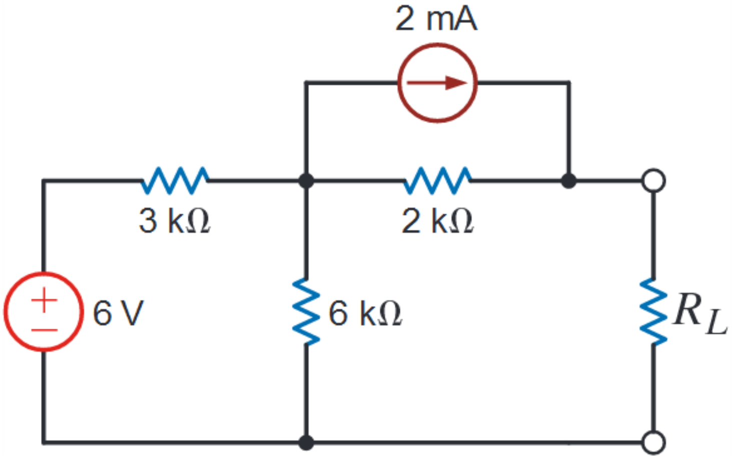
1- Find the output voltage for the following circuit. (30 points)



2- Find the Io for ther following circuit. (30 points)



3- Find (a) the value of load resistance for maximum power transfer and (b) the maximum power that can be transferred to the load in the following circuit. (2 x 20 points)

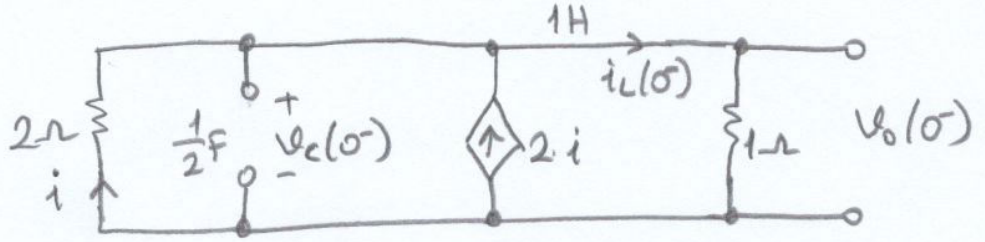


P.S. The duration is 105 minutes.

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
t	$\frac{1}{s^2}$
$\frac{t^n}{n!}$	$\frac{1}{s^{n+1}}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\frac{t^n e^{-at}}{n!}$	$\frac{1}{(s+a)^{n+1}}$
$\sin bt$	$\frac{b}{s^2+b^2}$
$\cos bt$	$\frac{s}{s^2+b^2}$
$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$
$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$

$f(t)$	$F(s)$
$Af(t)$	$AF(s)$
$f_1(t) \pm f_2(t)$	$F_1(s) \pm F_2(s)$
$f(at)$	$\frac{1}{a}F\left(\frac{s}{a}\right), a > 0$
$f(t-t_0)u(t-t_0), t_0 \geq 0$	$e^{-t_0 s}F(s)$
$f(t)u(t-t_0)$	$e^{-t_0 s}\mathcal{L}[f(t+t_0)]$
$e^{-at}f(t)$	$F(s+a)$
$\frac{d^n f(t)}{dt^n}$	$s^n F(s) - s^{n-1}f(0) - s^{n-2}f^1(0) \cdots - s^0 f^{n-1}(0)$
$tf(t)$	$-\frac{dF(s)}{ds}$
$t^n f(t)$	$(-1)^n \frac{d^n F(s)}{ds^n}$
$\frac{f(t)}{t}$	$\int_s^\infty F(\lambda)\, d\lambda$
$\int_0^t f(\lambda)\, d\lambda$	$\frac{1}{s}F(s)$
$\int_0^t f_1(\lambda) f_2(t-\lambda)\, d\lambda$	$F_1(s)F_2(s)$

(1) For $t<0$, to find initial values



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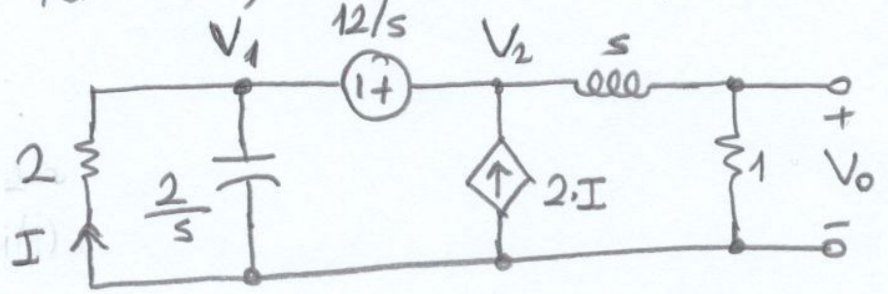
There is no independent source in the circuit.
Hence, $v_c(0^-)=0\,V$ and $i_L(0^-)=0\,A$.

For $t\geq 0$, the Laplace equivalent circuit



$$I=-\frac{V_1}{2} \qquad V_2-V_1=\frac{12}{s} \qquad V_o=\frac{V_2}{s+1}\cdot 1$$

$$V_1=V_2-\frac{12}{s}$$

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Node equation for supernode of V_1 and V_2:

$$\frac{V_1}{2}+\frac{V_1}{2/s}+(-2\cdot I)+\frac{V_2}{s+1}=0$$

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$$\frac{V_1}{2}+\frac{s\cdot V_1}{2}-2\cdot\left(-\frac{V_1}{2}\right)+\frac{V_2}{s+1}=0$$

$$V_1\cdot\left(\frac{1}{2}+\frac{s}{2}+1\right)+V_2\left(\frac{1}{s+1}\right)=0$$

$$\left(V_2-\frac{12}{s}\right)\cdot\left(\frac{3+s}{2}\right)+V_2\cdot\left(\frac{1}{s+1}\right)=0$$

→

1 continued

$$V_2 \cdot \left(\frac{s+3}{2} + \frac{1}{s+1} \right) - \frac{12}{s} \cdot \frac{s+3}{2} = 0$$

$$V_2 \cdot \frac{s^2+4s+5}{2 \cdot (s+1)} = \frac{6 \cdot (s+3)}{s}$$

$$V_2 = \frac{12 \cdot (s+1) \cdot (s+3)}{s \cdot (s^2+4s+5)}$$

$$V_0 = \frac{V_2}{s+1} = \frac{12 \cdot (s+3)}{s \cdot (s^2+4s+5)}$$

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$$V_0 = \frac{12 \cdot (s+3)}{s \cdot ((s+2)^2+1^2)} = \frac{A}{s} + \frac{B \cdot (s+2) + C}{s^2+4s+5}$$

$$12 \cdot (s+3) = A \cdot (s^2+4s+5) + s \cdot \left(B(s+2)+C \right)$$

$$12s+36 = A \cdot s^2 + 4 \cdot A \cdot s + 5 \cdot A + B \cdot s^2 + 2 \cdot B \cdot s + C \cdot s$$

$$A+B=0$$

$$4A+2B+C=12$$

$$5 \cdot A = 36 \Rightarrow A = \frac{36}{5} = 7{,}2$$

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$$B = -A = -7{,}2$$

$$C = 12 - 4A - 2B = -2{,}4$$

$$V_0 = \frac{7{,}2}{s} - 7{,}2 \cdot \frac{s+2}{(s+2)^2+1^2} - 2{,}4 \cdot \frac{1}{(s+2)^2+1^2}$$

→

1/continued

$$v_o(t) = 7{,}2 \cdot u(t) - 7{,}2 \cdot e^{-2t} \cdot \cos t \cdot u(t) - 2{,}4 \cdot e^{-2t} \cdot \sin t \cdot u(t)$$

5 $$v_o(t) = 7{,}2 \cdot u(t) + \sqrt{7{,}2^2 + 2{,}4^2} \cdot e^{-2t} \cdot \cos\left(t + \tan^{-1}\left(-\frac{-2{,}4}{-7{,}2}\right)\right) \cdot u(t)$$

$$v_o(t) = \left(7{,}2 + 7{,}58 \cdot e^{-2t} \cdot \cos(t + 161{,}57^\circ)\right) \cdot u(t)$$

(2) $V_2 - 0 = -6 \Rightarrow V_2 = -6V$ (5)

$V_4 - 0 = 12 \Rightarrow V_4 = 12V$ (5)

$V_1 - V_3 = 12 \Rightarrow V_3 = V_1 - 12 \Rightarrow V_1 = V_3 + 12$ (5)

Node equation for supernode of V_1 and V_3:

(5) $$\frac{V_1 - V_2}{2000} + \left(\frac{V_3 - V_2}{1000} + \frac{V_3}{2000} + \frac{V_3 - V_4}{1000}\right) + \frac{V_1 - V_4}{2000} = 0$$

$I_0 = \dfrac{V_3}{2000}$; so, to find V_3 is enough to solve question.

$$\frac{(V_3+12)-(-6)}{2000} + \left(\frac{V_3-(-6)}{1000} + \frac{V_3}{2000} + \frac{V_3-(12)}{1000}\right) + \frac{(V_3+12)-(12)}{2000} = 0$$

(5) $$V_3 + 18 + 2\cdot(V_3+6) + V_3 + 2\cdot(V_3-12) + V_3 = 0$$

$$7\cdot V_3 + 6 = 0$$

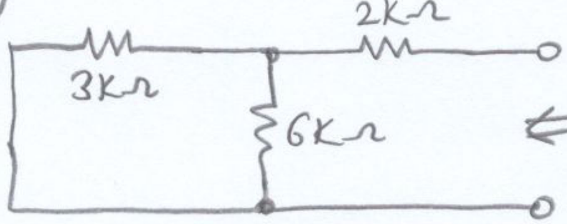
$$V_3 = -\frac{6}{7}\,V$$

(5) $$I_0 = \frac{V_3}{2000} = -\frac{3}{7}\ mA$$

③ Thanks to the maximum power transfer theorem,

$R_L = R_{Th}$ for transferring maximum power to the load. 10

ⓐ



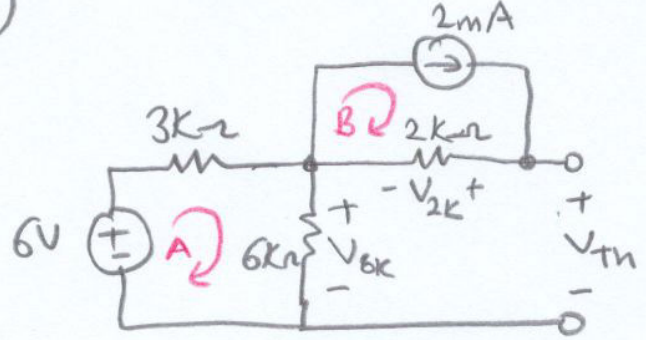
$$\Leftarrow R_{Th} = 2K\Omega + 3K\Omega // 6K\Omega$$

$$= 2K\Omega + 2K\Omega$$

$$= 4K\Omega \quad 10$$

So, $R_L = R_{Th} = 4K\Omega$

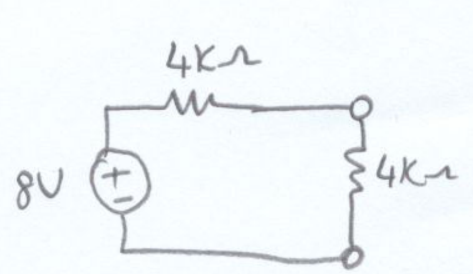
ⓑ



For loop A, $V_{6K} = \dfrac{6}{3000+6000} \cdot 6000 = 4V$ thanks to voltage-divider rule

For loop B, $V_{2K} = (2\,mA) \cdot (2K\Omega) = 4V$ thanks to Ohm's Law.

$V_{Th} = V_{2K} + V_{6K} = 8V$ 10



$$P_L = I_L^2 \cdot R_L = \frac{V_L^2}{R_L}$$

$$V_L = \frac{8}{4000+4000} \cdot 4000 = 4V$$

$$P_L = \frac{4^2}{4000} = 4\,mW \quad 10$$