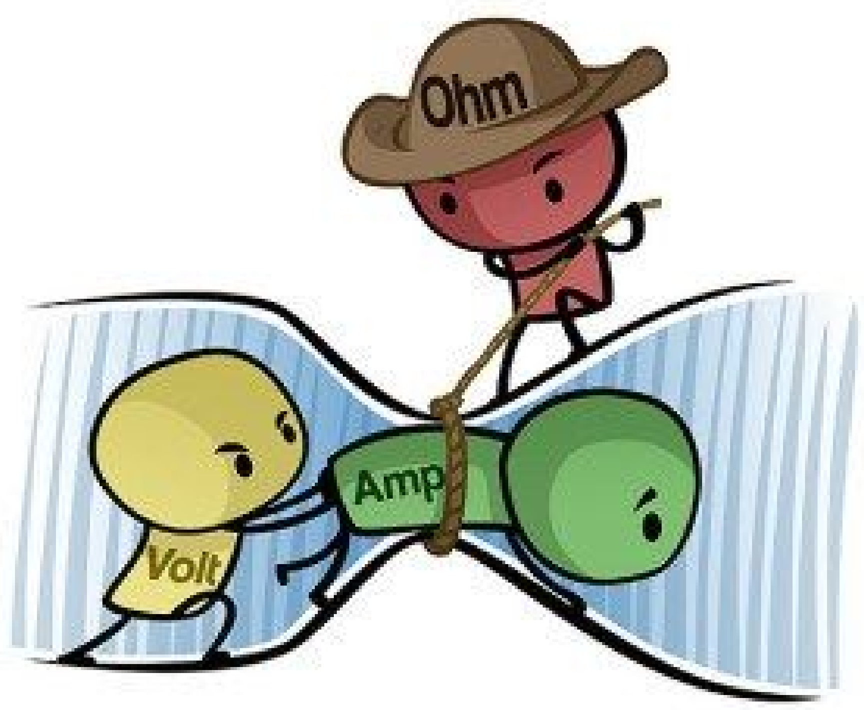
Electronic Circuits

Lecture 2.1: Basic Circuit Elements & Laplace (s) Domain Analysis

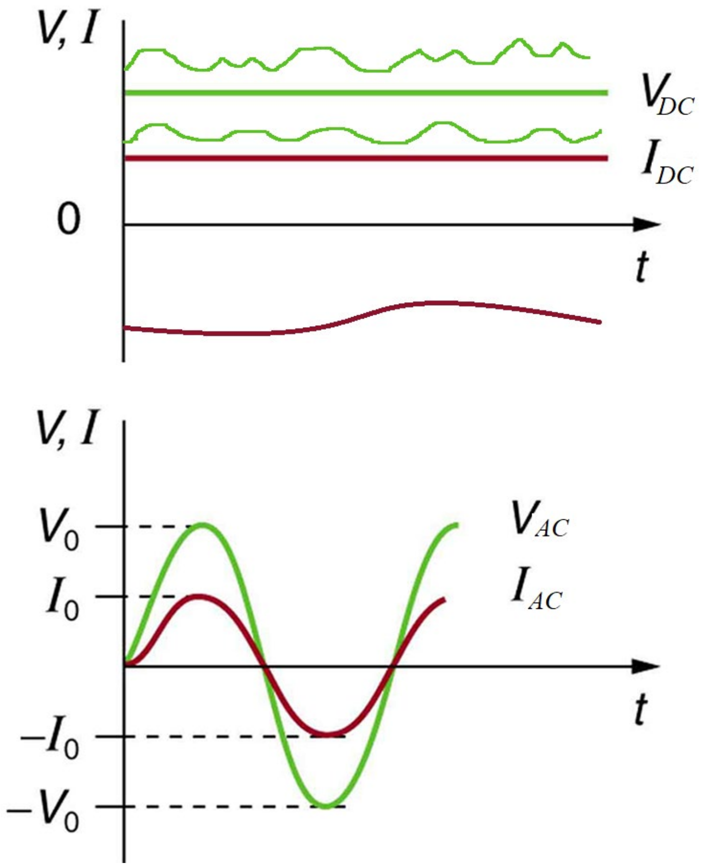
Electrical Quantities

- Conductors are materials let electricity flow through them easily. Most metals are good conductors.
- Insulators are materials don't carry electricity well. We need insulators to protect people from flowing electricity. Rubber and plastics are good examples.
- Voltage (V) is the force that makes electrons flow. It's a difference in potential energy between two different points in a circuit. It's measured in volts (V).
- Current (I) is the rate of the flow of electrons. It's measured in amperes (amps, A).
- Power (P) indicates instantenous work capacity. It is measured in watts (W) and calculated by V x I.
- Resistance (R), reactance (X), and impedance (Z) are the measures of how a material resists to conduct electricity. A low resistance indicates a good conductor, a high resistance means a good insulator. It is measured in ohms (Ω).



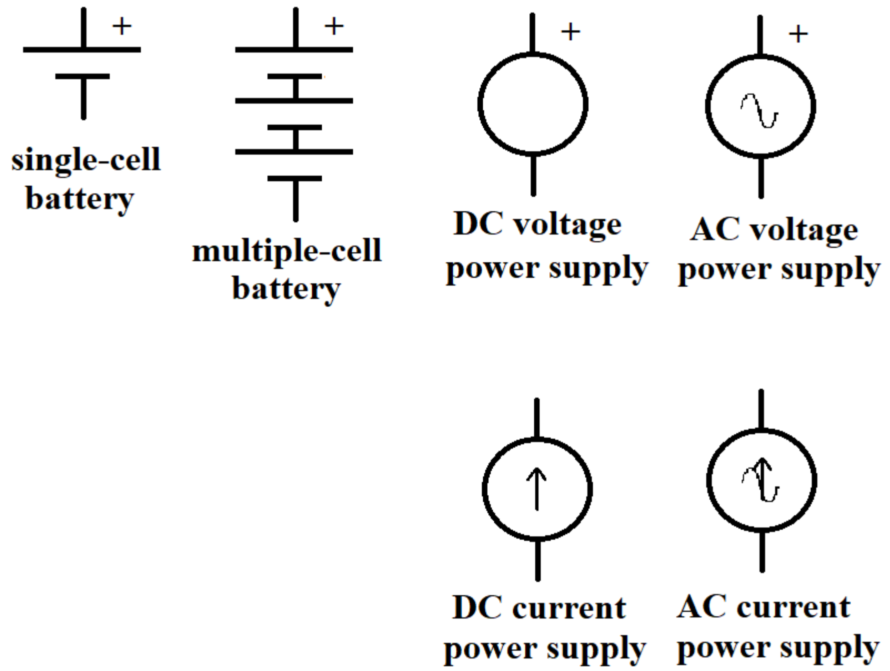
Sources (DC versus AC)

- A Source is a device which converts mechanical, chemical, thermal or some other form of energy into electrical energy.
- The source can supply either voltage (V) or current (I) at given certain value (or formula) by giving other one in free form.
- In direct current (DC), the electric charge (current) only flows in one direction. Electric charge in alternating current (AC), on the other hand, changes direction periodically.



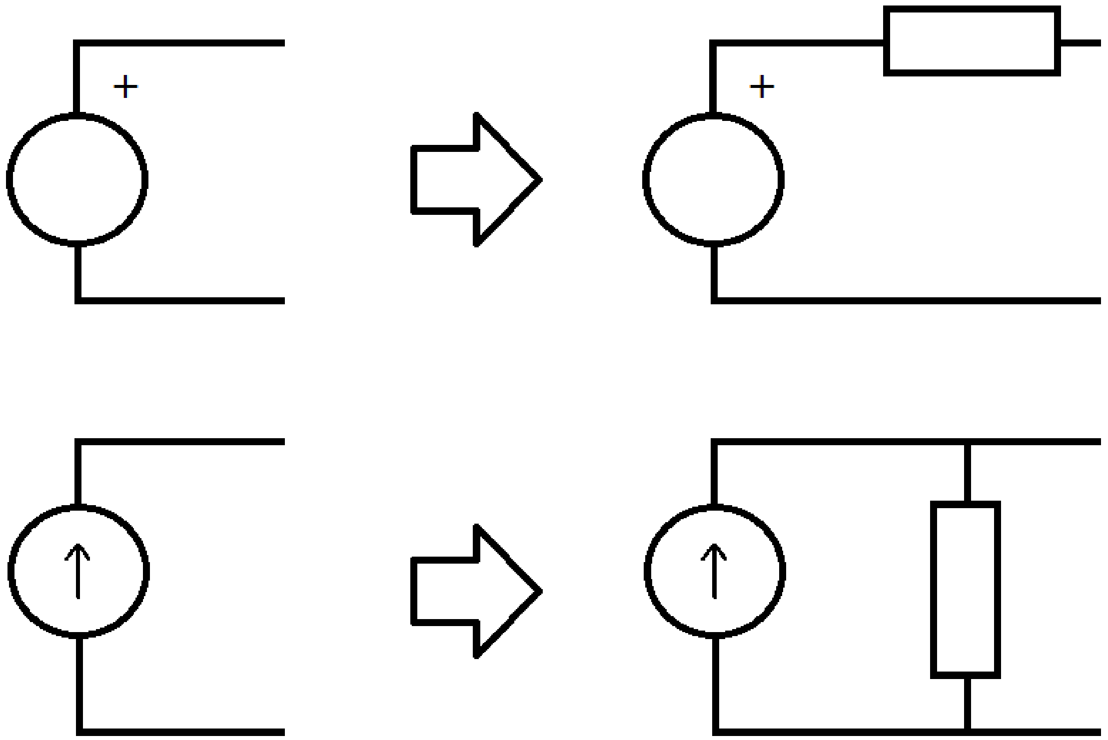
Sources (Voltage versus Current)

- An ideal voltage source is a two-terminal device whose voltage at any instant of time is constant and is independent of the current drawn from it.
- An ideal current source is a two-terminal circuit element which supplies the same current to any load resistance connected across its terminals.
- They have different types depending on whether their values are dependent (or controlled) to the other component’s electrical quantities:
  - Independent Voltage Source
  - Voltage-Dependent Voltage Source (VCVS)
  - Current-Dependent Voltage Source (CCVS)
  - Independent Current Source
  - Voltage-Dependent Current Source (VCCS)
  - Current-Dependent Current Source (CCCS)



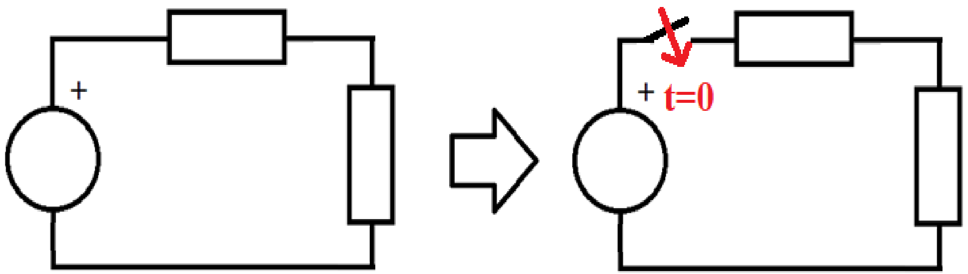
Sources (Ideal versus Practical)

- A practical voltage source is represented as an ideal voltage source connected with the small resistance in series.
- A practical current source is represented as an ideal current source connected with the high resistance in parallel.



Switches

- A switch changes its status open to close or vice versa at a specific time point.
- If there is no switch given in the circuit, we can assume that there is a switch closed at t=0.
- For this example, these two circuits are same for us. The switch had been opened from $t = -\infty$ to $t = 0^{-}$ (just before $t = 0$) and closed at $t = 0$.

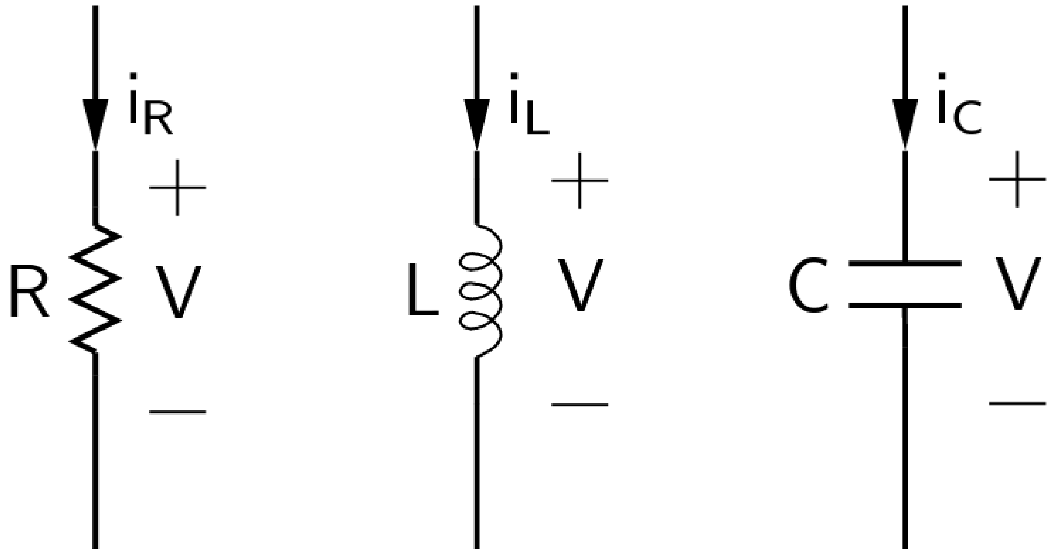


Voltage-Current Relations for R-L-C

$$i_R = \frac{1}{R} V \qquad V_R = i\,R$$

$$i_L = \frac{1}{L}\int_0 V\,dt + \mathbf{I_0} \qquad V_L = L\,\frac{di}{dt}$$

$$i_C = C\,\frac{dV}{dt} \qquad V_C = \frac{1}{C}\int_0 i\,dt + \mathbf{V_0}$$



V = I x R is called Ohm's Law.

Resistors in Laplace (s) Domain

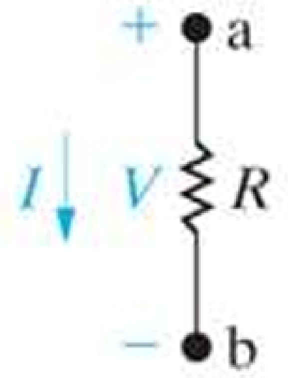
Laplace transform – resistors:

Time-domain

$$v(t) = Ri(t)$$

$$\xrightarrow{\mathcal{L}}$$

s-domain (Laplace)

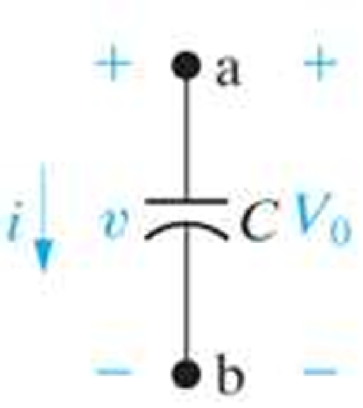


$$V(s) = RI(s)$$

Capacitors in Laplace (s) Domain

Laplace transform – capacitors

Time-domain



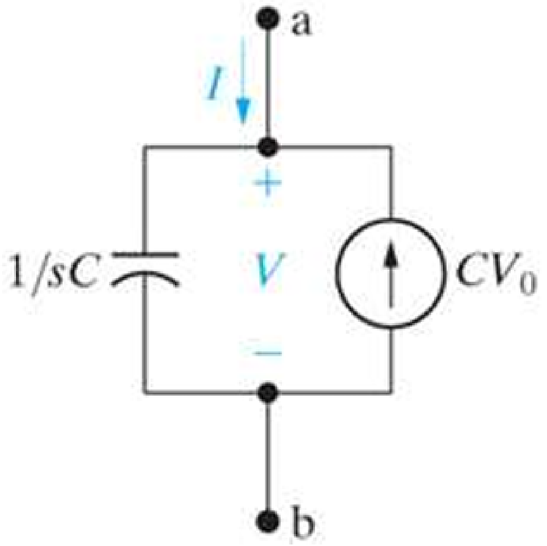
$$i(t) = C\frac{dv(t)}{dt}$$

$$v(0) = V_0$$

$$\mathcal{L}$$

$$\rightarrow$$

s-domain (Laplace)

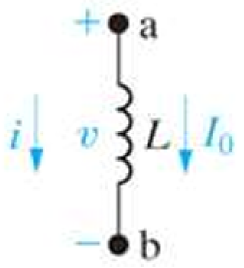


$$I(s) = sCV(s) - CV_0$$

Inductors in Laplace (s) Domain

Laplace transform – inductors:

Time-domain

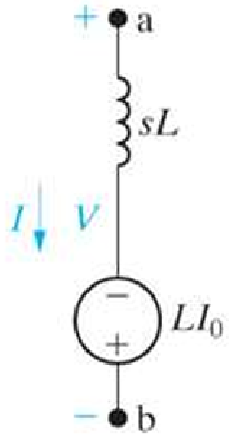


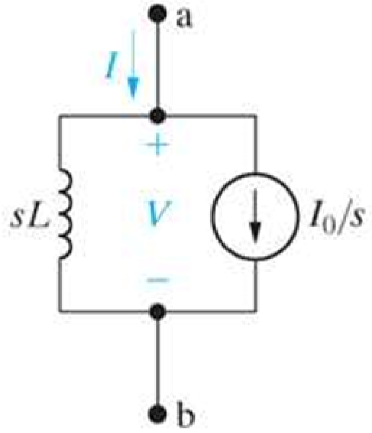
$$v(t) = L\frac{di(t)}{dt}$$

$$i(0) = I_0$$

$$\xrightarrow{\mathcal{L}}$$

s-domain (Laplace)





$$V(s) = sLI(s) - LI_0$$

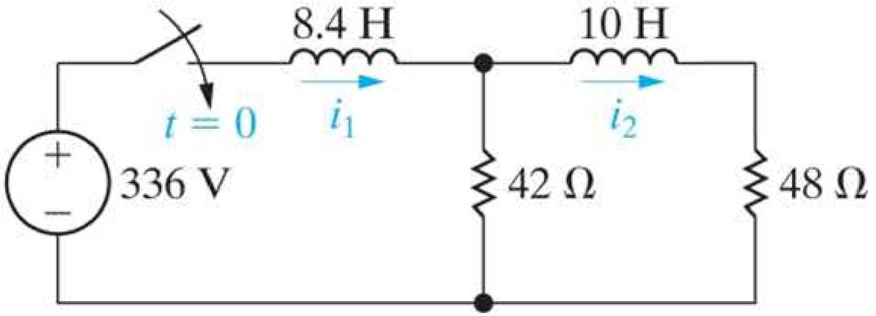
$$I(s) = \frac{V(s)}{sL} + \frac{I_0}{s}$$

Recipe For Laplace Transform Circuit Analysis

- Redraw the circuit (nothing about the Laplace transform changes the types of elements or their interconnections)
- Any voltages or currents with values given are Laplace-transformed using the functional and operational tables.
- Any voltages or currents represented symbolically, using $i(t)$ and $v(t)$, are replaced with the symbols $I(s)$ and $V(s)$.
- All component values are replaced with the corresponding complex impedance, $Z(s)$.
- Use DC circuit analysis techniques to write the s-domain equations and solve them. Check your solutions with initial value theorem (IVT) and final value theorem (FVT).
- Inverse-Laplace transform s-domain solutions to get time-domain solutions. Check your solutions at $t=0$ and $t=\infty$.

Example Analysis in Laplace (s) Domain

Example:
There is no initial energy stored in this circuit. Find $i_1(t)$ and $i_2(t)$ for $t > 0$.



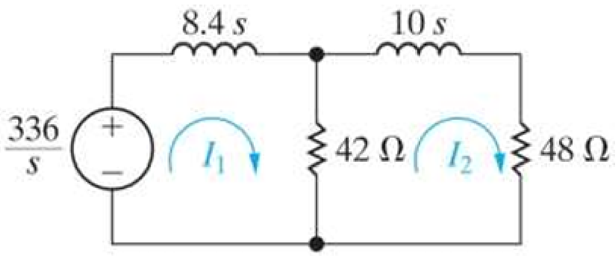
$$-\frac{336}{s} + (42 + 8.4s)\,I_1 - 42\,I_2 = 0$$

$$(10s + 90)\,I_2 - 42\,I_1 = 0 \qquad \Rightarrow \qquad I_1 = \frac{10s + 90}{42}\,I_2$$

Substituting, $$-\frac{336}{s} + \left[\frac{(42 + 8.4s)(10s + 90)}{42} - 42\right] I_2 = 0$$

$$\Rightarrow \qquad I_2(s) = \frac{336(42)}{s[(42 + 8.4s)(10s + 90) - 42^2} = \frac{168}{s^3 + 14s^2 + 24s}$$

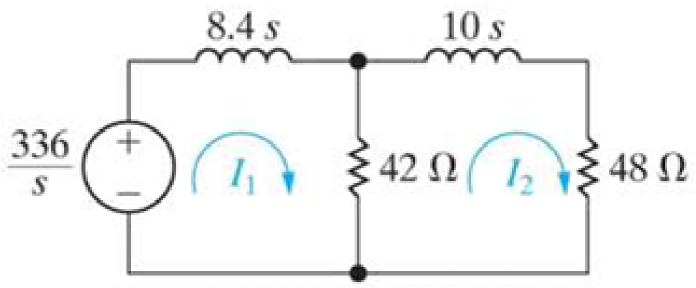
$$I_1(s) = \frac{10s + 90}{42}\left[\frac{168}{s^3 + 14s^2 + 24s}\right] = \frac{40s + 360}{s^3 + 14s^2 + 24s}$$



Example Analysis in Laplace (s) Domain cont'd

$$I_1(s) = \frac{40s+360}{s(s+2)(s+12)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s+2} + \frac{K_3}{s+12}$$



$$K_1 = \left.\frac{40s+360}{(s+2)(s+12)}\right|_{s=0} = 15; \qquad K_2 = \left.\frac{40s+360}{s(s+12)}\right|_{s=-2} = -14; \qquad K_3 = \left.\frac{40s+360}{s(s+2)}\right|_{s=-12} = -1$$

$$\therefore \qquad I_1(s) = \frac{15}{s} + \frac{-14}{s+2} + \frac{-1}{s+12}$$

$$i_1(t) = \mathcal{L}^{-1}\left\{\frac{15}{s} + \frac{-14}{s+2} + \frac{-1}{s+12}\right\}$$

$$= [15 - 14e^{-2t} - e^{-12t}]u(t) \text{ A}$$

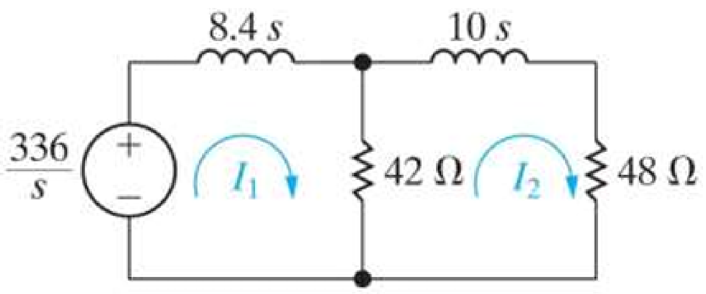
The forced response is $15u(t)$ A;

The natural response is $[-14e^{-2t} - e^{-12t}]u(t)$ A.

Example Analysis in Laplace (s) Domain cont'd

$$I_2(s) = \frac{168}{s(s+2)(s+12)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s+2} + \frac{K_3}{s+12}$$



$$K_1 = \left.\frac{168}{(s+2)(s+12)}\right|_{s=0} = 7; \qquad K_2 = \left.\frac{168}{s(s+12)}\right|_{s=-2} = -8.4; \qquad K_3 = \left.\frac{168}{s(s+2)}\right|_{s=-12} = 1.4$$

$$\therefore \qquad I_2(s) = \frac{7}{s} + \frac{-8.4}{s+2} + \frac{1.4}{s+12}$$

$$i_2(t) = \mathcal{L}^{-1}\left\{\frac{7}{s} + \frac{-8.4}{s+2} + \frac{1.4}{s+12}\right\}$$

$$= [7 - 8.4e^{-2t} + 1.4e^{-12t}]u(t) \text{ A}$$

The forced response is $7u(t)$ A;

The natural response is $[-8.4e^{-2t} - 1.4e^{-12t}]u(t)$ A.

Example Analysis in Laplace (s) Domain: Initial Value Check

$$I_1(s) = \frac{40s + 360}{s^3 + 14s^2 + 24s}$$

$$\lim_{t\to 0} i_1(t) = \lim_{s\to\infty} sI_1(s)$$

$$= \lim_{s\to\infty} \frac{40s^2 + 360s}{s^3 + 14s^2 + 24s}$$

$$= \lim_{1/s\to 0} \frac{(40/s) + (360/s^2)}{1 + (14/s) + (24/s^2)}$$

$$= 0 \text{ A(check!)}$$

$$I_2(s) = \frac{168}{s^3 + 14s^2 + 24s}$$

$$\lim_{t\to\infty} i_1(t) = \lim_{s\to\infty} sI_1(s)$$

$$= \lim_{s\to\infty} \frac{168s}{s^3 + 14s^2 + 24s}$$

$$= \lim_{1/s\to 0} \frac{(168/s^2)}{1 + (14/s) + (24/s^2)}$$

$$= 0 \text{ A(check!)}$$

Example Analysis in Laplace (s) Domain: Final Value Check

$$I_1(s) = \frac{40s + 360}{s^3 + 14s^2 + 24s}$$

$$\lim_{t\to\infty} i_1(t) = \lim_{s\to 0} sI_1(s)$$

$$= \lim_{s\to 0} \frac{40s^2 + 360s}{s^3 + 14s^2 + 24s}$$

$$= \lim_{s\to 0} \frac{40s + 360}{s^2 + 14s + 24}$$

$$= \frac{360}{24} = 15\ \mathrm{A(check!)}$$

$$I_2(s) = \frac{168}{s^3 + 14s^2 + 24s}$$

$$\lim_{t\to\infty} i_1(t) = \lim_{s\to 0} sI_1(s)$$

$$= \lim_{s\to 0} \frac{168s}{s^3 + 14s^2 + 24s}$$

$$= \lim_{s\to 0} \frac{168}{s^2 + 14s + 24}$$

$$= \frac{168}{24} = 7\ \mathrm{A(check!)}$$

Example Analysis in Laplace (s) Domain: t=0 Check

At $t = 0$, the circuit has no initial stored energy, so $i_1(0) = 0$ and $i_2(0) = 0$. Now check the equations:

$$i_1(0) = (15 - 14 - 1)(1) = 0$$

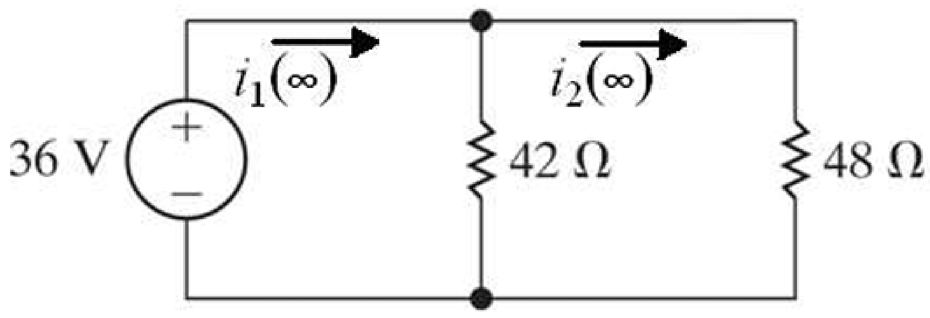
$$i_2(0) = (7 - 8.4 + 1.4)(1) = 0$$

Example Analysis in Laplace (s) Domain: t=∞ Check

$$i_1(t) = (15 - 14e^{-2t} - e^{-12t})\,u(t)\ A \quad \Rightarrow \quad i_1(\infty) = 15 - 0 - 0 = 15\ \text{A}$$

$$i_2(t) = (7 - 8.4e^{-2t} + 1.4e^{-12t})\,u(t)\ A \quad \Rightarrow \quad i_2(\infty) = 7 - 0 - 0 = 7\ \text{A}$$

Draw the circuit for t = ∞ and check these solutions.



$$42 \,||\, 48 = 22.4\Omega$$

$$i_1(\infty) = \frac{336}{22.4} = 15\ \text{A(check!)}$$

$$i_2(\infty) = \frac{22.4}{48}(15) = 7\ \text{A(check!)}$$

LTS: Left To Students (Not homework, try yourself)

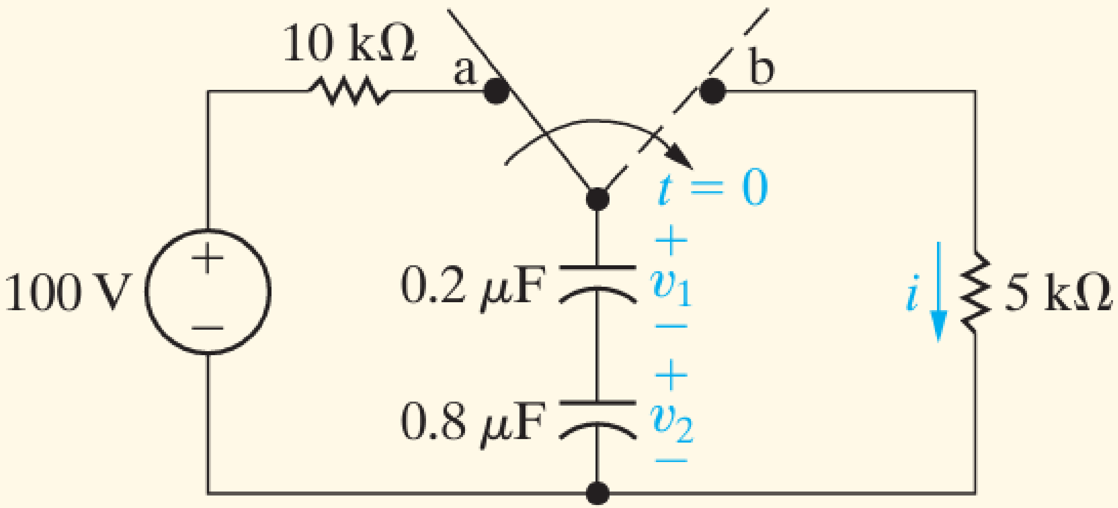
The switch in the circuit shown has been in position a for a long time. At $t = 0$, the switch is thrown to position b.

a) Find I, V_1, and V_2 as rational functions of s.

b) Find the time-domain expressions for i, v_1, and v_2.

Answer: (a) $I = 0.02/(s + 1250)$,
$V_1 = 80/(s + 1250)$,
$V_2 = 20/(s + 1250)$;

(b) $i = 20e^{-1250t}u(t)$ mA,
$v_1 = 80e^{-1250t}u(t)$ V,
$v_2 = 20e^{-1250t}u(t)$ V.



Thanks for listening ☺

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