FILTERS

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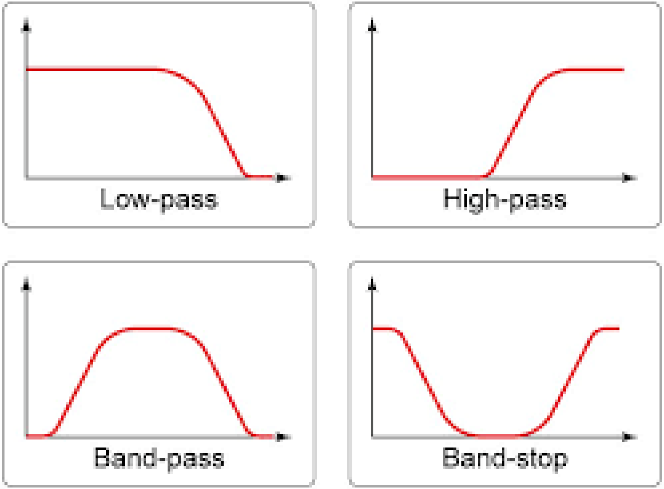
WHAT IS FILTER

- Filter is a device,either software or hardware, that is used to removal of unwanted signal and noise and to attenuate certain signal frequency ranges and highlight other frequency regions.
- There are two broad classes of filter;namely analog filters for continuous signals and digital filters,which filter discrete signals .

- Also,filters can be analyzed in 4 categories according to working principles as low pass filter,high pass filter,band-pass,band-stop filter.



ANALOG FILTERS

Analog filters are electronic circuit built from component such as resistors, capacitors,and inductors.

- The main aspect in filter design is filter response,which is characterized by its transfer function,H(s),and on Bode plots.The transfer function is written in terms of magnitude and phase:

$$H(e^{jw}) = |H(e^{jw})| < \phi(w) \tag{1}$$

- $s = \sigma + jw$

$$H(s) = \frac{b_0 + b_1 s + ... + b_m s^m}{a_0 + a_1 + ... + a_n s^n} \tag{2}$$

- a_i and b_j are scalar coefficients

Pole-zero equation

$$H(s) = \frac{N(s)}{D(s)} = K\frac{(s - z_1)(s - z_2)...(s - z_m)}{(s - p_1)(s - p_2)...(s - p_n)} \tag{3}$$

z_i are the zeroes or roots of the equation

$$N(s) = 0 \tag{4}$$

p_i are the poles or roots of denominator

$$D(s) = 0 \tag{5}$$

The gain of the system is denoted by K

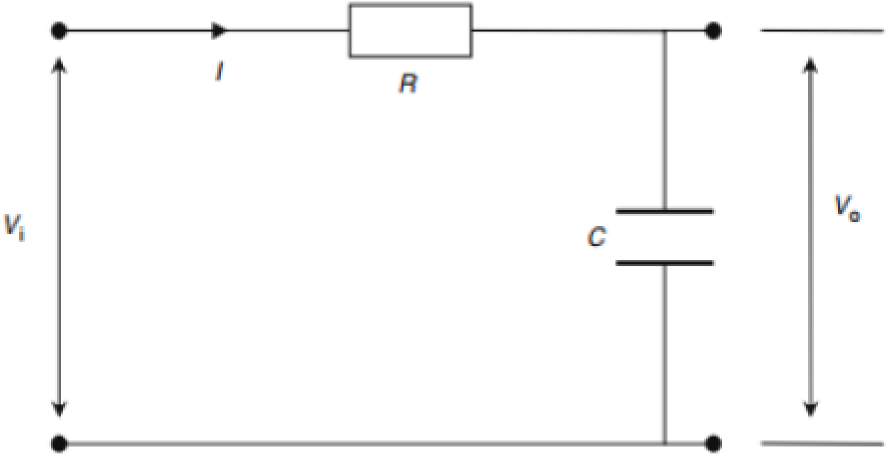


Figure Filter characteristics and region indicating the comprise between ideal filters and practical filter

- Three basic analog electronic components used in the construction of analog filters are resistor (R),capacitor(C),and inductor(I).
- Using combination of (R),(C),and (I) it is possible to design simple passive filters as well as more complex filters by combining these component either in series or in parallel as RC,RL,LC,RLC circuits.

RC FILTER

The impedance of a resistor is measured in ohms where as complex impedance of a capacitor is written as

$$Z_c = \frac{1}{sC} \tag{6}$$

C is the capacitance measured in Farads. $S = a + jw$ when a=0

$$Z_c = \frac{1}{jwC} \tag{7}$$

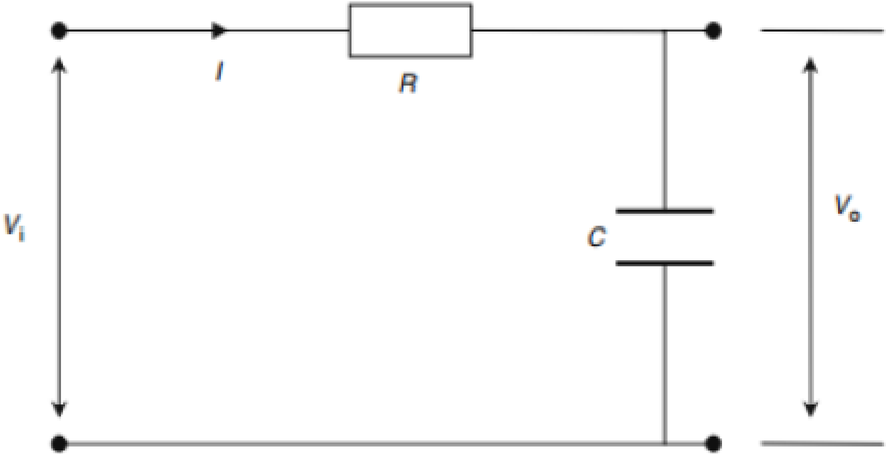


Figure Circuit diagram for RC low pass filter

- The transfer function is defined as the ratio of the output voltage V_0 to the input voltage V_i with the output measured across the capacitor.

$$\frac{V_0}{V_i} = \frac{\frac{1}{sC}}{R + (\frac{1}{sC})} = \frac{1}{1 + sRC} \tag{8}$$

$$Gain = |H(w)| = |\frac{V_0}{V_i}| = \frac{1}{\sqrt{1 + (wRC)^2}} \tag{9}$$

- The phase angle of the output voltage across the capacitor as

$$\phi_c = tan^{-1}(-wRC) \tag{10}$$

Characteristic of low pass filter

- Analyze the gain relationship as input signal frequency w varies.When w is small or w approach the 0,
- We substituting w=0 into Equation 9 gives gain unity $|H(w)| = 1$
- When w is large or w approach the ∞,
- we substituting $w = \infty$ into Equation 9 gives $|H(W)| = 0$
- when input signal frequency is $w = \frac{1}{RC}$ We find gain

$$|H(w)| = \frac{1}{\sqrt{2}} \tag{11}$$

- Taking the log_{10} of this gives us the gain in decibels or dB as

$$|H(w)| = -20log_{10}\frac{1}{\sqrt{2}} = -3dB \tag{12}$$

- We define the frequency at which the filter gain falls by 3 dB as the filter cutoff frequency, w_c.

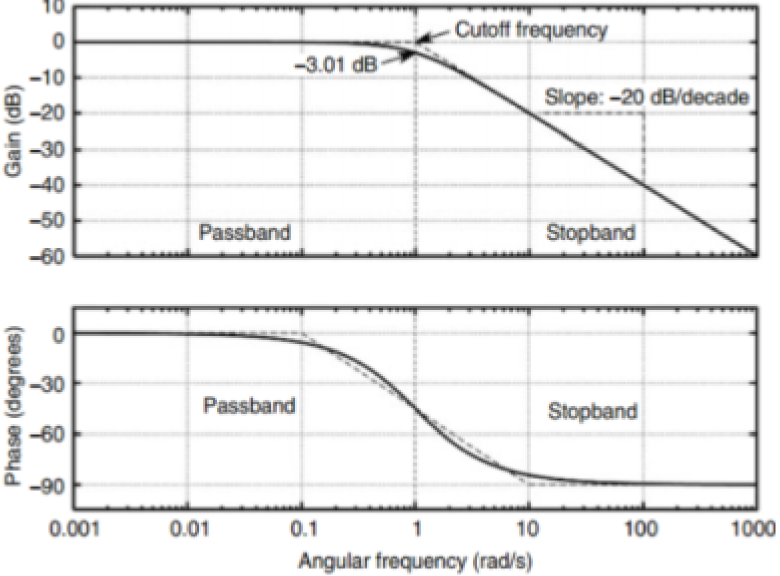


Figure Magnitude and phase variation over frequency range of input signal for a RC low-pass filter.

RL Filter

nductor can be used to design a high-pass filter circuit. The complex impedance of an inductor is given as

$$Z_L = sL \tag{13}$$

L is the inductance measured in Henry. Inductor impedance can be written as $Z_L = jwL$.

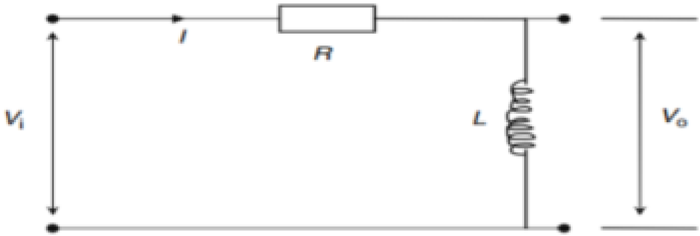


Figure Circuit diagram for RL high pass filter.

RL Filter

$$\frac{V_0}{V_i} = \frac{jwL}{R + jwL} \tag{14}$$

$$|H(W)| = |\frac{V_0}{V_i}| = \frac{wL}{\sqrt{R^2 + w^2 L^2}} \tag{15}$$

- Phase angle of output voltage taken across the inductor is then

$$\phi_L = tan^{-1}(\frac{R}{wL}) \tag{16}$$

- Butterworth filters are higher-order filters with the special characteristic that as filter order increase,the filter response approaches that of an ideal filter(Akay,1994).
- Where the attenuation region become sharper as the filter order increase.
- Filter order or number of L and C (N) and the cutoff frequencies (w_c) are important issues to design a Butterworth filter

General magnitude of the Butterworth filters response

$$|H(w)|^2 = \frac{1}{1 + (\frac{w}{w_c})^{2N}} \tag{17}$$

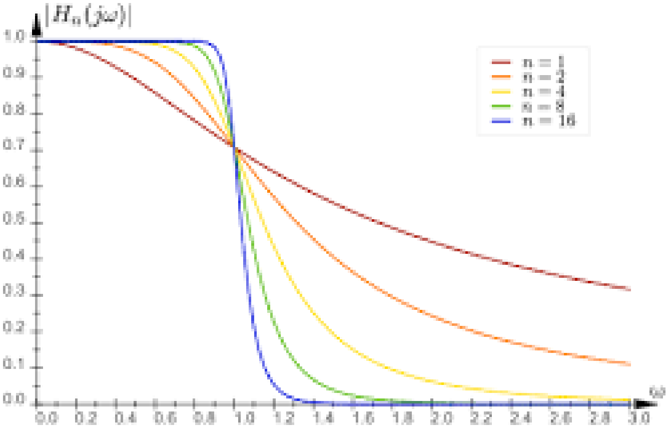


Figure A typical frequency response of an nth order Butterworth filter.

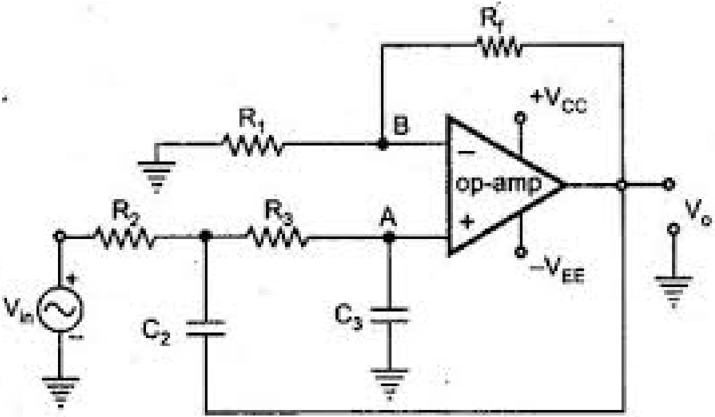


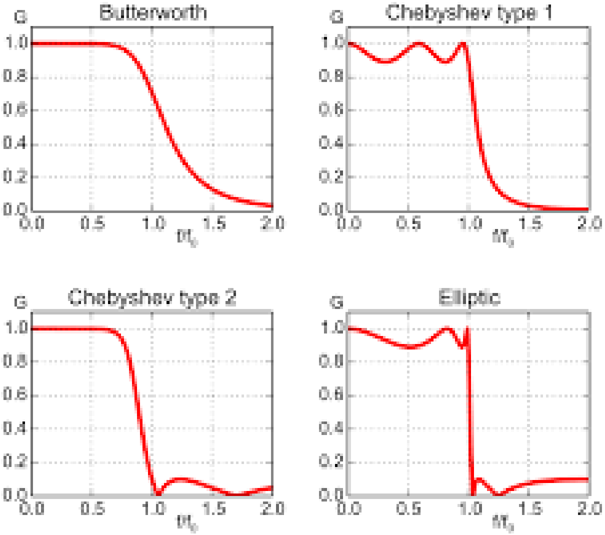
Fig. 2.76 Second order low pass butterworth filter

$$A_p = 10log_{10}[1 + (\frac{w_p}{w_c})^{2N}] \tag{18}$$

$$A_s = 10log_{10}[1 + (\frac{w_s}{w_c})^{2N}] \tag{19}$$

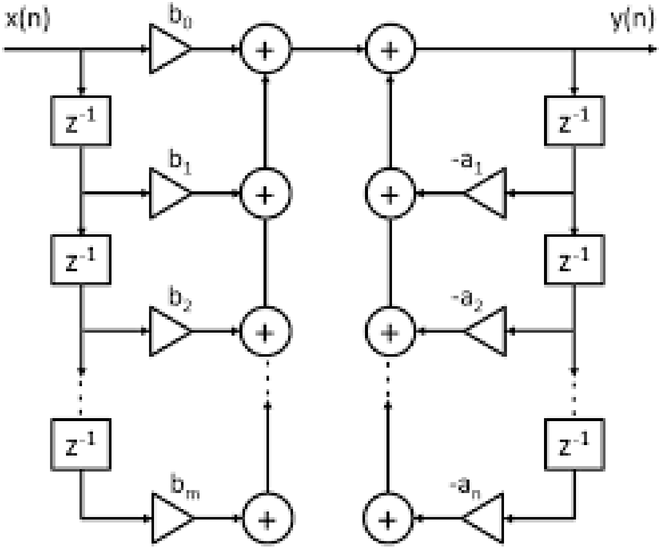
A_p is the maximum passband attenuation, A_s the minimum stopband attenuation, w_p the passband frequency, and w_s the stopband frequency.

- Chebyshev filters have steeper attenuation region than Butterworth filters at the expense of greater rippling effect in passband region.
- There are two type Chebyshev filters as Type-I and Type-II Chebyshev filters.

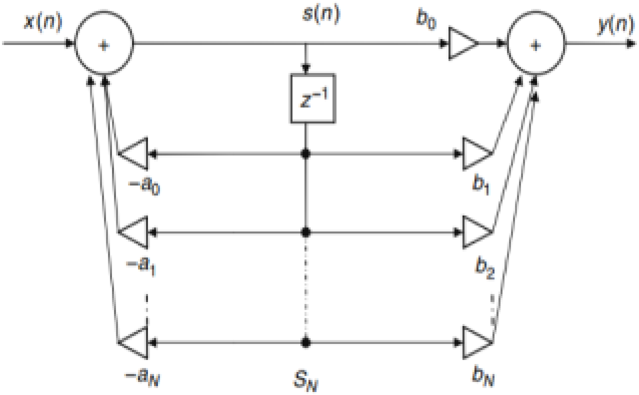

Butterworth
Chebyshev type 1
Chebyshev type 2
Elliptic
G
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0

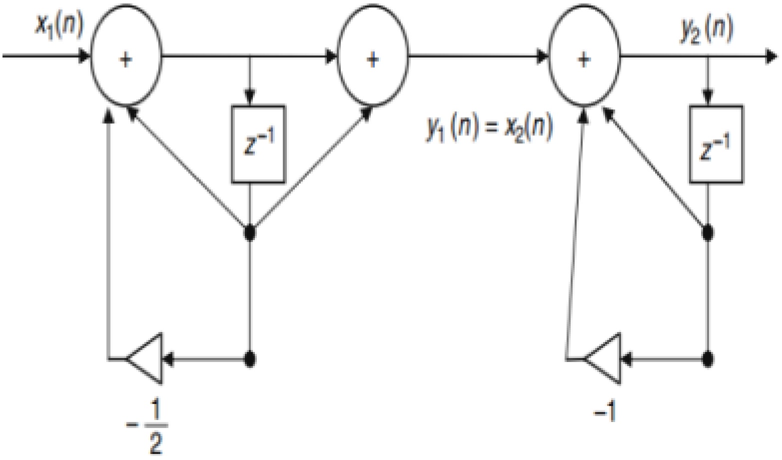
In signal processing, a digital filter is a system that performs mathematical operations on a sampled, discrete-time signal to reduce or enhance certain aspects of that signal.

- Two classes of digital filters are Finite Impulse Response (FIR) and Infinite Impulse Response (IIR). The term 'Impulse Response' refers to the appearance of the filter in the time domain. The mathematical difference between the IIR and FIR implementation is that the IIR filter uses some of the filter output as input.


x(n)
b0
y(n)
z-1
b1
-a1
z-1
z-1
b2
-a2
z-1
z-1
bm
-an

Canonical IIR Filter




$x_1(n)$
+
+
+
$y_2(n)$
z^{-1}
z^{-1}
$y_1(n) = x_2(n)$
$-\frac{1}{2}$
-1

- In this method, the impulse response of the analog filter is first sampled equally to obtain the digital impulse response. This is written as

$$h_D(n) = h_A(nT) \tag{20}$$

- where T is the sampling period, and hD(n) the digital response obtained from the analog response hA(nT). Suppose that the analog transfer function in the time domain is given as

$$h_A(t) = \sum_{p=1}^{N} A_p e^{s_p t} u(t) \tag{21}$$

- where u(t) is the unit step. This can be transformed to the Laplace equation as

$$H_A(s) = \sum_{p=1}^{N} \frac{A_p}{s - s_p} \tag{22}$$

- Now using Equation 20, we see that the digital response can be similarly obtained as follows:

$$h_D(n) = h_A(nT) = \sum_{p=1}^{N} A_p e^{n s_p T} u(nT) \tag{23}$$

- The digital transform is written using the z-transform giving

$$H_D(z) = \sum_{p=1}^{N} \frac{A_p}{1 - e^{s_p T} z^{-1}} \tag{24}$$

- Writing the z-transform of the digital transfer function as a Laplace transform, we obtain where $z = e^{sT}$, $w = \Omega T$, and is the analog frequency whereas the digital frequency. If the analog filter is bandlimited, the digital filter function takes the form

$$H(z) = \frac{1}{T} H_A(j\frac{w}{T}) \tag{25}$$

- where $|w| =< \pi$is the cutoff frequency. At this point we have

$$H_A(j\frac{w}{T}) = 0 \tag{26}$$

- $|\frac{w}{T}| >= \frac{\pi}{T}$

- Another method for deriving digital filters from analog counterparts is the bilinear transformation. This method begins with the assumption that the derivative of the analog signal is obtainable, that is, we have

$$\frac{d_y(t)}{dt} = x(t) \tag{27}$$

$$sY(s) = X(s) \rightarrow \frac{Y(s)}{X(s)} = \frac{1}{s} \tag{28}$$

$$\int_{(n-1)T}^{nT} \frac{dy(t)}{dt} = y(nT) - y[(n-1)T] \tag{29}$$

$$= \int_{(n-1)T}^{nT} x(t) \tag{30}$$

Design of IIR Digital Filters

Bilinear Transformation

$$y(nT) - y[(n-1)T] = \frac{T}{2}(x(nT) + x[(n-1)T]) \tag{31}$$

$$y(n) - y(n-1) = \frac{T}{2}[x(n) + x(n-1)] \tag{32}$$

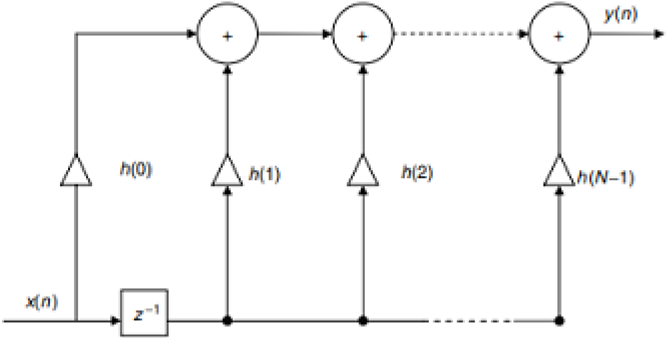
$$H(z) = \frac{Y(z)}{X(z)} = \frac{T}{2}\frac{1+z^{-1}}{1+z^{-1}} \tag{33}$$

$$s = \frac{2}{T}\frac{1+z^{-1}}{1+z^{-1}} \tag{34}$$

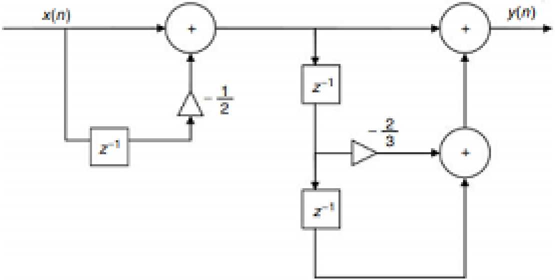
- The bilinear transformation is preferable to the previous method because it does not suffer from anti-aliasing effects, though it sometimes results in oscillatory behavior in the resulting digital design. This method is frequently used to derive higher-order filters.

- In FIR filters, the output is determined using only the current and previous input samples. The output then has the following form:

$$y(n) = \sum_{j=0}^{M} \frac{b_j}{a_0} x(n-j) \tag{35}$$



$$y(n) = \sum_{j=0}^{M} h(j)x(n-j) \tag{36}$$



$$H(z) = (1 - \frac{1}{2}z^{-1})(1 - \frac{2}{3}z^{-1} + z^{-2}) \quad (37)$$

- There are several well-known methods for designing FIR filters such as window, frequency sampling, minmax, and optimal design techniques. We will review just two of these methods, namely, the window and frequency sampling methods.

- Using this method, an FIR filter can be obtained by selecting a set of infinite duration impulse response sequences.Given an ideal frequency response $H_I(e^{jw})$, the corresponding impulse response $h_I(n)$ can be estimated using the following integral:

$$h_1(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_I(e^{jw}) e^{jwn} dw \quad (38)$$

$$H_I(e^{jw}) = \sum_{n=-\infty}^{\infty} h_I(n) e^{jwn} \quad (39)$$

- if n between 0 and $N-1$ h(n) get $h_I(n)$ otherwise g it get 0 value.
- h(n) can be represented by the product of the infinite impulse response $h_I(n)$ and a window $w(n)$

- There are six popular types of window , which are listed here.

1. *Rectangular*. For $0 \le n \le N-1$

$$w(n) = 1$$

2. *Bartlett*

$$w(n) = \begin{cases} \dfrac{2n}{N-1} & 0 \le n \le \dfrac{N-1}{2} \\ 2 - \dfrac{2n}{N-1} & \dfrac{N-1}{2} \le n \le N-1 \end{cases}$$

3. *Hanning*. For $0 \le n \le N-1$, the window is defined as

$$w(n) = \frac{1}{2}\left[1 - \cos\left(\frac{2\pi n}{N-1}\right)\right]$$

This is also referred to as the raised cosine window because it is symmetric.

Design of FIR Filters
The Window Method

- There are six popular types of window , which are listed here.

4. *Hamming*. For $0 \leq n \leq N-1$, the window is defined as

$$w(n) = 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right)$$

This window is also known as the raised cosine platform, which has N nonzero terms.

5. *Blackman*. For $0 \leq n \leq N-1$, the window function is written as

$$w(n) = 0.42 - 0.5\cos\left(\frac{2\pi n}{N-1}\right) + 0.08\left(\frac{4\pi n}{N-1}\right)$$

- One advantage of the Kaiser window is increased design flexibility, the Kaiser parameter α can be used to control the main lobe widths and the side band ripple

6. *Kaiser.* For $0 \le n \le N-1$, the window function is written as

$$w(n) = w_{\mathrm{R}}(n) I_0 \frac{\alpha\sqrt{1-(n/N)^2}}{I_0(\alpha)}$$

where $w_{\mathrm{R}}(n)$ is the rectangular window, α the Kaiser parameter, and I_0 a modified Bessel function of the first kind of order zero, which has the following form:

$$I_0(\alpha) = \sum_{m=0}^{\infty} \left[\frac{\alpha}{m!2^m}\right]^2$$

- The frequency sampling method is another popular technique for designing FIR filters. First, the discrete Fourier transform is employed to represent the coefficients of the frequency response. Then the impulse response coefficients are determined via the inverse discrete Fourier transform. The discrete Fourier representation for a frequency response $H(k)$ can be written as

$$H(k) = H_I(e^{jw}) \tag{40}$$

- where the samples have frequency W= 2 πk/N for k = 0, 1, 2,...,N-1.Then the filter coefficients are derived using the inverse transform via

$$h(n) = \frac{1}{N} \sum_{k=-1}^{N-1} H(k) e^{\frac{j2\pi nk}{N}} \tag{41}$$

- for n = 0, 1, 2,...,N 1. Since the FIR coefficients take real values they can be estimated from all the complex terms in the complex conjugate pairs, so that we have

$$h(n) = \frac{1}{N}\left(H(0) + 2\sum k = 1^{\frac{N-1}{2}} \mathrm{Re}\left[H(k)\mathrm{e}^{\frac{j2\pi nk}{N}}\right]\right) \quad \text{if } N \text{ is odd}$$
$$h(n) = \frac{1}{N}\left(H(0) + 2\sum k = 1^{\frac{N}{2}-1} \mathrm{Re}\left[H(k)\mathrm{e}^{\frac{j2\pi nk}{N}}\right]\right) \quad \text{if } N \text{ is even}$$

$$H(z) = \sum_{n=0}^{N-1} h(n)z^{-n} \tag{42}$$

- The frequency sampling method is therefore a direct method compared to the window technique because it avoids transformations from the time domain to the frequency domain.

- Integer filters are another form of digital filter that are primarily deployed in environments requiring fast online processing. The previous digital filters can be implemented on computer software, however, the floating point operations performed on the real coefficients of the transfer function limit somewhat the speed of computation. In integer filters, these coefficients are replaced by integers making the computations faster and more efficient by using integer arithmetic operations. These operations require only bit shifting operations rather than the slower floating point unit (FPU) for computations. Such filtering is especially desirable for high-frequency digital signals or when computers have slow microprocessors. The major limitation of the integer filter is that it becomes difficult to obtain sharp cutoff frequencies by using only integer coefficients in the filter transfer function.

- General form of transfer function

$$H_1(z) = \frac{[1 - z^{-m}]^p}{[1 - 2\cos(\theta)z^{-1} + z^{-2}]^t}$$

$$H_2(z) = \frac{[1 + z^{-m}]^p}{[1 - 2\cos(\theta)z^{-1} + z^{-2}]^t}$$

- Placement of poles

$$\text{Denominator} = (z - e^{j\theta})(z - e^{-j\theta})$$

Multiplying the two factors, we get

$$\text{Denominator} = z^2 - (e^{j\theta} + e^{-j\theta})z + (e^{j\theta}e^{-j\theta})$$

Using the identity,

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

We arrive at

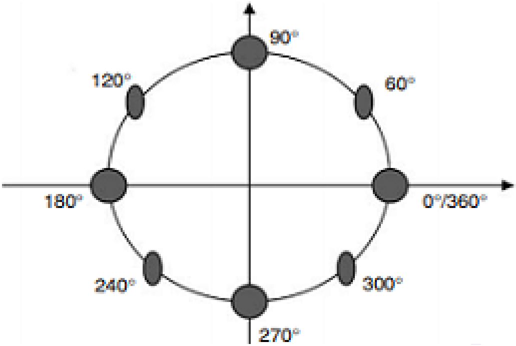
$$\text{Denominator} = 1 - 2\cos(\theta)z^{-1} + z^{-2}$$

- Placement of zeros

$$(1 - z^{-m}) \tag{43}$$

$$(1 + z^{-m}) \tag{44}$$

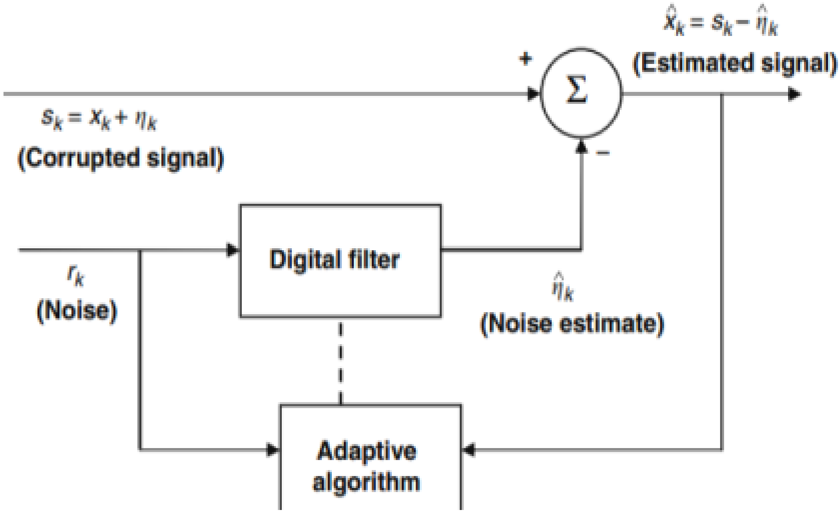
- Possible pole zero placement on the unit circle in the z plane for integer filter design.



ADAPTIVE FILTERS

- An adaptive filter is a digital filter with self-adjusting characteristics.
- It adapts automatically, to changes in its input signals.
- A variety of Adaptive algorithms have been developed for the operation of adaptive filters, e.g., LMS , RLS, etc. *LMS (least Mean Square) *RLS (Recursive Least Squares)
- Contains 2 main component : 1- Digital filter(with adjustable coefficients). 2- Adaptive Algorithm.

ADAPTIVE FILTERS



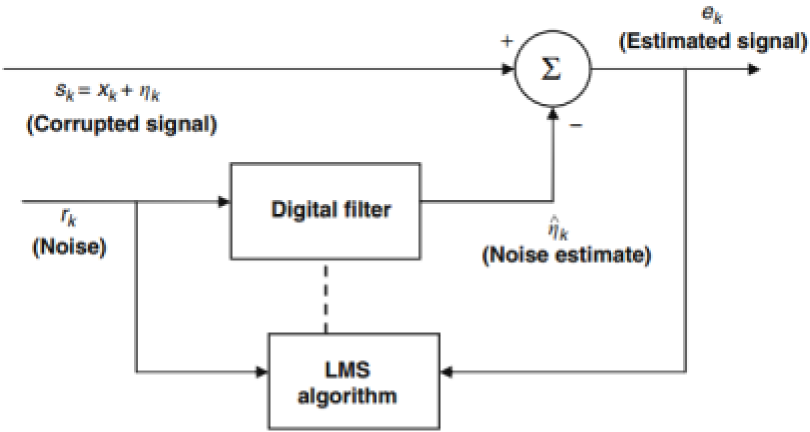


Figure Adaptive filter with coefficient determined using the least mean squares algorithm.

The LMS Algorithm

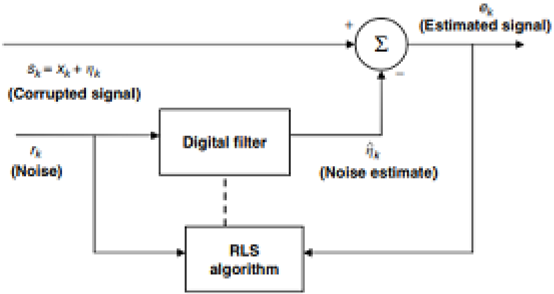
LMS ADVANTAGES

- Simplicity of implementation
- Not neglecting the noise like Zero forcing equalizer
- Stable and robust performance against different signal conditions

LMS DISADVANTAGES

- Slow Convergence
- Demands using of training sequence as reference

The Recursive Least Squares (RLS) ALGORITHM



- Adaptive filter with coefficients determined using the recursive least squares algorithm.

The Recursive Least Squares (RLS) ALGORITHM

$$\varepsilon(n) = \sum_{i=1}^{n} \underbrace{\beta(n,i)}_{\text{weight factor}} |e(i)|^2 \qquad 0 < \beta(n,i) \le 1$$

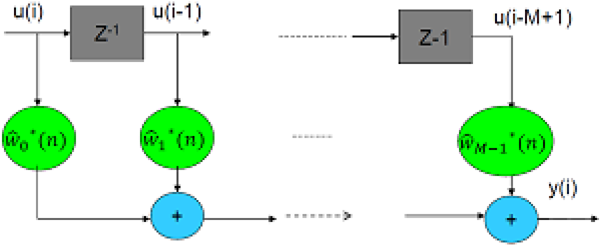
$$e(i) = d(i) - y(i) = d(i) - w^H(n)u(i) = d(i) - \sum_{k=0}^{M-1} w_k(n)u(i-k)$$

$$u(n) = [u(i) \quad u(i-1) \ldots\ldots\ldots u(i-M+1)]^T$$

$$w(n) = [w_0(i) \quad w_1(i-1) \ldots\ldots\ldots w_{M-1}(i-M+1)]^T$$

- Define the time-dependent cost function $\epsilon(n)$

The Recursive Least Squares (RLS) ALGORITHM



- Least square solution $\phi(n)\hat{w}(n) = p(n)$

- Recursion for correlation matrix

$$\underbrace{\emptyset(n)}_{\boxed{\text{New value}}} = \lambda \underbrace{\emptyset(n-1)}_{\boxed{\text{Previous value}}} + \underbrace{u(n)u^H(n)}_{\boxed{\text{correction}}}$$

$$p(n) = \lambda p(n-1) + u(n)d^*(n)$$

$\hat{w}(n) = \emptyset^{-1}(n)p(n)$ matrix inversion ☹ ☹

The Recursive Least Squares (RLS) ALGORITHM

- To avoid $\emptyset^{-1}(n)$ calculation use the relation (Let A and B be positive definite MxM matrices

$$A = B^{-1} + CD^{-1}C^{H} \longrightarrow$$

D; NxM
C; MxN matrices

- Martix inversion lemma: $A^{-1} = B - BC(D + C^{H}BC)^{-1}C^{H}B$)

Now, $A = \emptyset(n)$, $B^{-1} = \lambda\emptyset(n-1)$, $C = u(n)$, $D = 1$

$$\emptyset(n) = \lambda\emptyset(n-1) + u(n)u^{H}(n) \qquad \emptyset(n)^{-1} = ?$$

The Recursive Least Squares (RLS) ALGORITHM

- $\emptyset(n)^{-1} = \lambda^{-1}\emptyset(n-1)^{-1} - \frac{\lambda^{-2}\emptyset^{-1}(n-1)u(n)u^{H}(n)\emptyset^{-1}(n-1)}{1+\lambda^{-1}u^{H}(n)\emptyset^{-1}(n-1)u(n)}$

$P(n) = \emptyset^{-1}(n)$; MxM inverse correlation matrix

$k(n) = \frac{\lambda^{-1}P(n-1)u(n)}{1+\lambda^{-1}u^{H}(n)P(n-1)u(n)}$, gain factor

$P(n) = \lambda^{-1}P(n-1) - \lambda^{-1}\, k(n)u^{H}(n)\, P(n-1)$, Ricot equation

$$\begin{aligned} k(n) &= [\lambda^{-1}P(n-1) - \lambda^{-1}\, k(n)u^{H}(n)\, P(n-1)]u(n) \\ &= P(n)u(n) = \emptyset^{-1}(n)u(n) \end{aligned}$$

The Recursive Least Squares (RLS) ALGORITHM

$$
\begin{aligned}
\hat{w}(n) &= \varnothing^{-1}(n)p(n) \\
&= P(n)p(n) = P(n)[\lambda p(n-1) + u(n)d^*(n)] \\
&= [\lambda^{-1}P(n-1) - \lambda^{-1}\, k(n)u^H(n)\, P(n-1)][\lambda p(n-1) + u(n)d^*(n)] \\
&= \hat{w}(n-1) - k(n)u^H(n)\hat{w}(n-1) + \underbrace{P(n)u(n)}_{k(n)}d^*(n)
\end{aligned}
$$

$$
\begin{aligned}
\hat{w}(n) &= \hat{w}(n-1) + \mathrm{k}(n)[d^*(n) - u^H(n)\hat{w}(n-1)] \\
&= \hat{w}(n-1) + \mathrm{k}(n)\varphi^*(n)
\end{aligned}
$$

$$\varphi(n) = d(n) - u^T(n)\hat{w}^*(n-1) = d(n) - \hat{w}^H(n-1)n(n), \text{ previous error}$$

Summary of the RLS Algorithm

$P(0) = \delta^{-1} I,$ I: identity matrix, δ: small positive constant

$\hat{w}(n) = 0,$ for n=1, 2, … compute

1. $k(n) = \dfrac{\lambda^{-1} P(n-1) u(n)}{1+\lambda^{-1} u^H(n) P(n-1) u(n)}$

2. $\varphi(n) = d(n) - \hat{w}^H(n-1) u(n)$

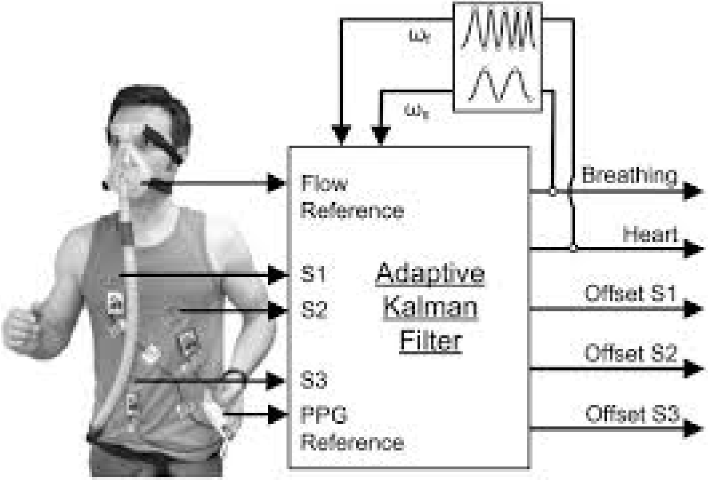
3. $\hat{w}(n) = \hat{w}(n-1) + \mathrm{k(n)} \varphi^*(n)$

4. $P(n) = \lambda^{-1} P(n-1) - \lambda^{-1} k(n) u^H(n) P(n-1)$ go back to (1)

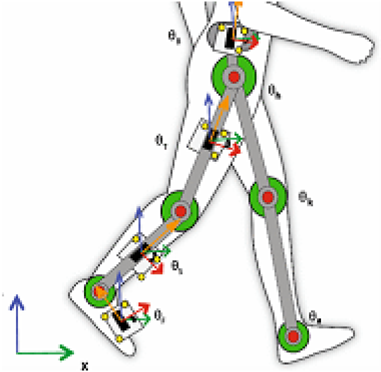
- A very popular approach that is time-varying is the Kalman filter.
- The Kalman filter is a recursive state space model based estimation algorithm.
- Kalman filter can use estimation of parameter in EEE signal processing.
- Kalman Filter can use to noise estimation during Electrocardiography.
- The extended Kalman filter as a pulmonary blood flow estimator
- Kalman filter approach to remove TMS(Transcranial Magnetic Situmutation) induced artifact from EEG recording is proceed.
- Kalman filter technique applied for medical image reconstruction.

- Kalman Filter approach for cardiorespiratory signal extraction an fusion of non-contracting sensors. so it is possible to measure in real time heart and breathing rates using an adaptive Kalman filter approach. Adapting the Kalman filter matrices improves the estimation result and makes the filter universally deplorable when measuring cardiorespiratory signal.

- An adaptive Kalman filter approach for cardiorespiratory signal extraction and fusion of non-contacting sensors



- Global Kalman filter approaches to estimate absolute angles of lower limb segments



- A Kalman filter technique applied for medical image reconstruction

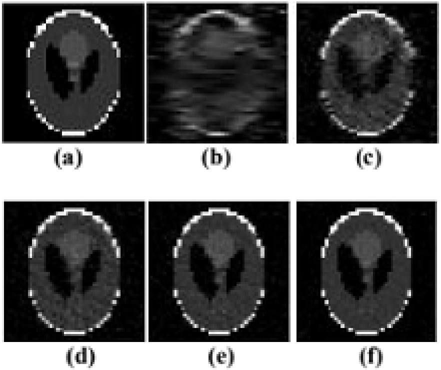


Figure 6. a) Original image and results of reconstruction from noisy projection using b) 5 projection c) 20 projection d) 35 projection e) 45 projection f) 60 projection

- Kalman filter can use estimation of parameter in EEG signal processing.

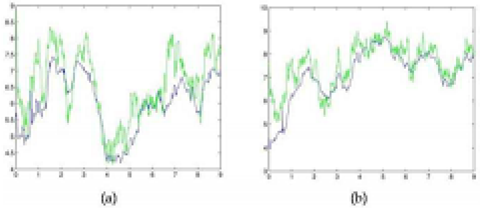
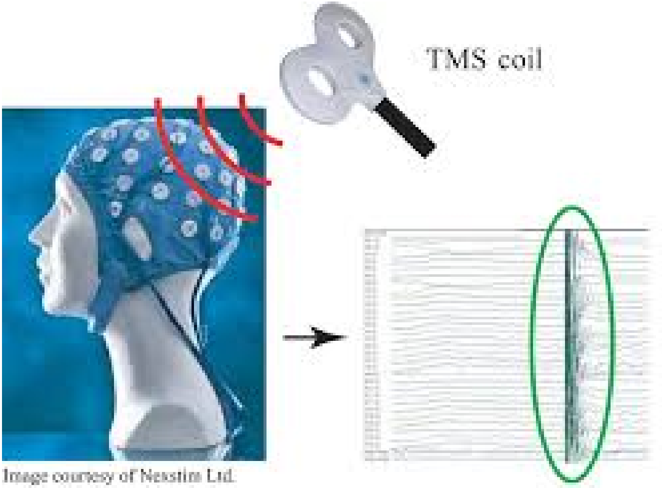
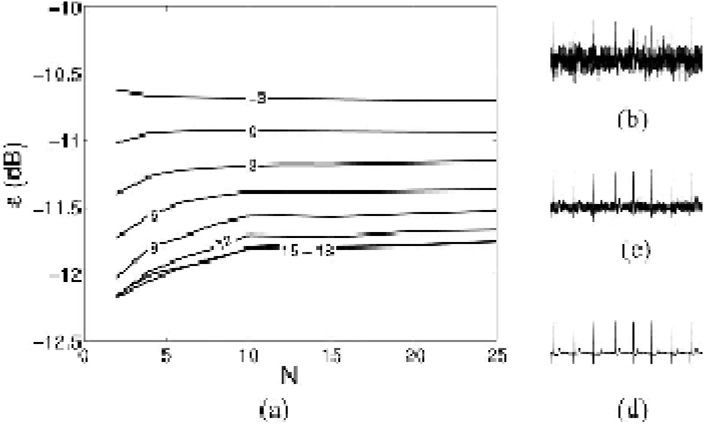


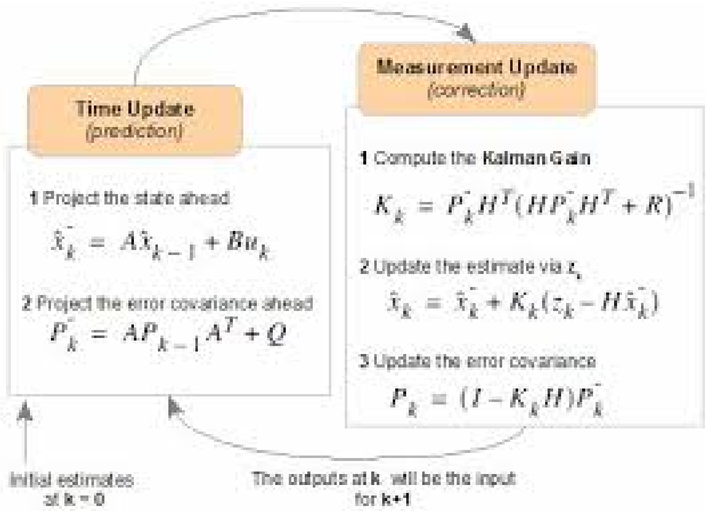
Fig. 10. Mean Instantaneous Frequency for right hand movement. (a) Channel C3. (b) Channel C4. In both figures the blue line corresponds to the Kalman Smoother with EM and green line to the Kalman Smoother only.

- Application of Kalman filter to remove TMS-induced artifacts from EEG recordings



- An Adaptive Kalman Filter for ECG Signal Enhancement





A complete picture of the operation of the Kalman Filter.

KALMAN FILTER

$$G(n) = F(n+1,n)K(n,n-1)C^H(n)[C(n)K(n,n-1)C^H(n) + Q_2(n)]^{-1}$$

$$\alpha(n) = y(n) - C(n)\hat{x}(n\backslash n-1)$$ innovation proces

$$\hat{x}(n+1\backslash n) = F(n+1,n)\hat{x}(n\backslash n-1) + G(n)\alpha(n)$$ State update

$$K(n) = K(n,n-1) - F(n,n+1)G(n)C(n)K(n,n-1)$$ Kalman Gain

$$K(n+1,n) = F(n+1,n)K(n)F^H(n+1,n)Q_1(n)$$ Ricatti equation

The End