BIOMEDICAL SIGNAL PROCESSING

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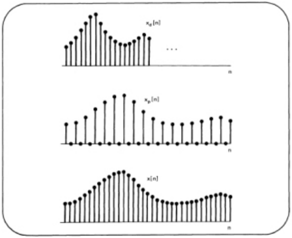
CONTENTS

Signals

Signal : A function that contains information that has one or more variables.



- Measuring and interpreting of biosignals is important in biomedical engineering.
- Electrical signals result from action potentials in our cells.
- The correct measurement and interpretation of signals is vital.

Signals and Signal Systems

A signal can be represented in both time and frequency domains.

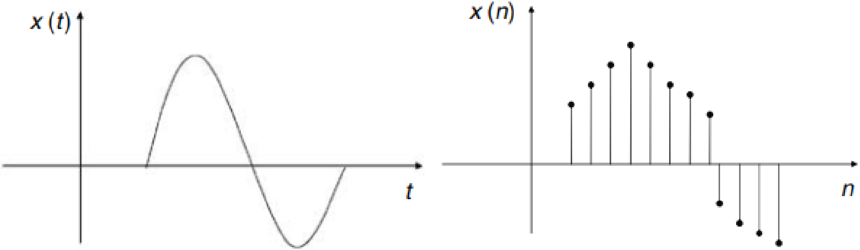


FIGURE 2.1
Left: A time continuous analog signal. Right: A discrete sequence signal.

Analog signal and Digital Signal

Analog Signal

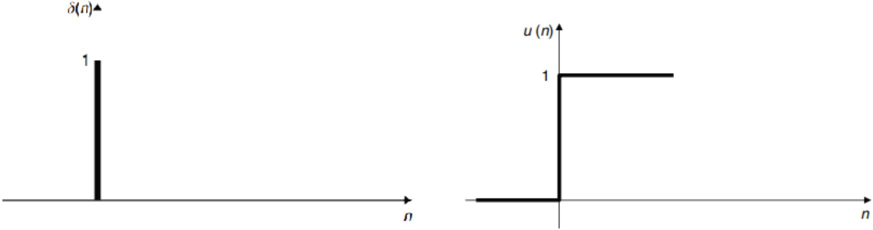
- Analog signal uninterrupted and continuous.
- It consists of endless points.

Digital Signal

- Digitized signals. So there are values of 1 and 0. (Binary)
- It is not continuous. Because it's numerical.
- It is cheaper than the analog signal.

Linear Shift Invariant Systems

There are many forms of deterministic or periodic signals; the two most common are the unit impulse function and the unit step function.



$$\delta(n) = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{for } n \neq 0 \end{cases} \qquad h(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

FIGURE 2.2 (a) A unit impulse function, δ(n). (b) A unit step function, u(n).

Linear Shift Invariant Systems

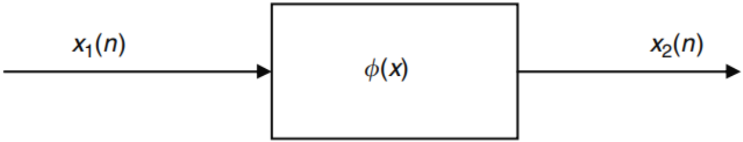


FIGURE 2.3
A signal system consisting of one input signal, which is transformed into the output.

The transformation operator known as the transfer function (Φ)

$$x_2(n) = \Phi[x_1(n)] \tag{1}$$

A system is said to be a linear system if there is principle of superposition:

$$\Phi[ax_1(n) + bx_1(n)] = a\Phi[x_1(n)] + \Phi b[x_1(n)] \tag{2}$$

Linear Shift Invariant Systems

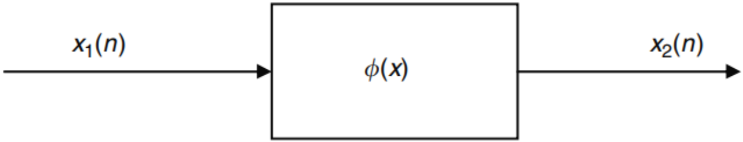


FIGURE 2.4
A signal system consisting of one input signal, which is transformed into the output.

A shift invariant system is a digital system;

$$x_1(\text{n}) \text{ to } x_2(\text{n}) \text{ and}$$
$$x_1(\text{n-d}) \text{ to } x_2(\text{n-d})$$

d is a delay or shift in the digital sequence.
The analog version of this system is the linear time invariant system.

Linear Shift Invariant Systems

The impulse response of a system be written as

$$h = \Phi(\delta_0) \tag{3}$$

Any output signal x_2(n) can be written in terms of the input signal x_1(n) as

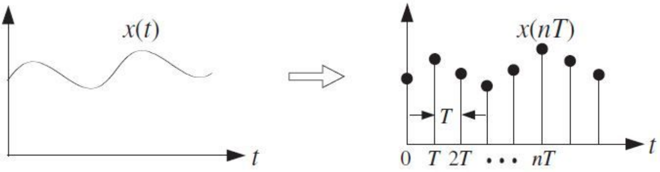
$$x_2(n) = \sum_{k=-\infty}^{\infty} x_1(k)\delta_0(n-k) \tag{4}$$

Then the output of a general linear shift invariant system can be represented by

$$x_2(n) = \sum_{k=-\infty}^{\infty} x_1(k)h(n-k) \tag{5}$$

Sampling and Analog Reconstruction

The reproducibility of the signal depends on how often the signal is sampled on the transmitter.



The sampling signal frequency (fsr) must be greater than twice the maximum frequency (fc) of the information signal.

$$(\text{fsr} > or = 2\text{fc})$$

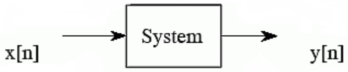
The minimum sampling rate is known as the Nyquist rate, where the Nyquist frequency is defined as

$$\text{fNR} = \text{fsr}/2$$

Sampling and Analog Reconstruction

- Analog reconstruction is the opposite of digitization with the aim to recover the original time continuous signal given a sequence of discrete values.
- To do this, the digital signal is low-pass filtered with cutoff frequency equal to the Nyquist frequency.
- This removes the higher frequency components and smoothens the gaps between digital samples.

Causality and Stability



CAUSALITY

A system is said to be 'causal' if the output at any time depends only on inputs up to that time and not after.

$$x_1[n] = x_2[n], n < n_0$$

$$x_1[n] \neq x_2[n], n > n_0$$

$$y_1[n_0] = y_2[n], n \in n_0$$

STABILITY

This implies that the outputs of the system are bounded for all bounded input sequences.

$$\max(|x[n]|) < \infty$$

$$\max(|y[n]|) < \infty$$

Noise

- Noise is often regarded as unwanted signals.
- Noise signals are "stationary" if the probabilistic characteristics of the signal do not change with time, for example, constant mean or variance.
- White noise refers to signals with a flat spectrum, which is constant and independent of frequency.
- Pseudorandom signals are generated using some complex and unknown algorithm.

Signal Transforms

In this section, we examine several different signal transforms.

- The Fourier Transform

  The Fourier transform allows the identification of non-periodic signals in the frequency domain.
  For a continuous time signal x(t), the Fourier transform is given by the integral X;

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \tag{6}$$

  and x(t) can be recovered by the inverse Fourier transform;

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega \tag{7}$$

- The Fourier Transform

  The Fourier transform for the digital sequence is then a summation of N;

$$X(k) = \sum_{n=1}^{N} x(n) e^{-j\omega n} \tag{8}$$

The inverse discrete Fourier transform is just;

$$x(n) = \frac{1}{N} \sum_{k=1}^{N} X(k) e^{j\omega n} \tag{9}$$

Signal Transforms

- Fast Fourier Transform
  Fast fourier transformation is often used in signal processing because it allows the frequencies of a signal to be measured. This transformation is a special case of the Fourier transform.
  -Fast fourier transformation makes Fourier Transform very fast within the algorithms.

- The Wavelet Transform
  - Wavelet transform decomposes a signal into a set of basis functions. These basis functions are called wavelets.
  - The wavelet analysis method is a time-frequency analysis method.
  - You can use this representation to characterize transient events, reduce noise, compress data, and perform many other operations.
  - Discrete wavelet transform (DWT), which transforms a discrete time signal to a discrete wavelet representation.

$$W(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t).\overline{\Psi(\frac{t-b}{a})} dt \tag{10}$$

$$\Psi_{(a,b)}(t) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} \Psi(\frac{t-b}{a}) \tag{11}$$

- The Wavelet Transform

$$W(a, b) = \int_{-\infty}^{\infty} x(t)\ \overline{\Psi_{(a,b)}(t)}dt \tag{12}$$

In these equations a $>$ 0, b $\in \Re$ a, scaling parameter; b conversion parameter; x (t) sign; Ψ, wavelet function (main wavelet); W (a, b) also sign indicates continuous wavelet transform.

- The Wavelet Transform
  Common applications of wavelet transforms include:
  - Speech and audio processing
  - Image and video processing
  - Biomedical imaging
  - 1D and 2D applications in communications and geophysics.

Signal Transforms

- The z-Transform
  z transformation is a more general form of discrete time Fourier Transform.
  The z transformation of discrete time f(n) signal is expressed as follows;

$$F(z) = \sum_{n=-\infty}^{\infty} f(n) z^{-n} \tag{13}$$

z is a complex variable and expressed as; z=$|z|e^{jn}$

The z-transform is useful for solving difference equations and also to derive the transfer function of the system.

Signal Transforms

- Discrete Cosine Transform (DCT)
  -DCT is often used in signal processing for compression purposes.
  -The DCT is a transform similar to the DFT, except that it uses only the real part of the signal. It transforms a signal or image from the spatial domain to the frequency domain.
  The standard DFT can be written as follows for k = 1, 2,...,N:

$$X(k) = \sum_{n=1}^{N} x(n) e^{(-j2\pi nk)/N} \tag{14}$$

and the DCT can be obtained by taking the transform of the real part giving;

$$X_c(k) = Re[\sum_{n=1}^{N} x(n) e^{(-j2\pi nk)/N}] \tag{15}$$

- Discrete Cosine Transform

$$X_c(k) = \sum_{n=1}^{N} x_n cos(\frac{2\pi nk}{N}) \tag{16}$$

A more common form of DCT is given by the following for all k = 1, 2,...,N:

$$X_c(k) = \frac{1}{N} \sum_{n=1}^{N} x_n cos(\frac{\pi k(2n+1)}{2N}) \tag{17}$$

The inverse of the DCT can be obtained using the following:

$$X_c(k) = \frac{1}{2} x_0 \sum_{n=2}^{N} x_n cos(\frac{\pi n(2k+1)}{2N}) \tag{18}$$

- The Discrete Walsh Transform
  - The Walsh-Hadamard transform is a non-sinusoidal, orthogonal transformation technique that decomposes a signal into a set of basis functions. These basis functions are Walsh functions, which are rectangular or square waves with values of $+1$ or –1.
  - Like the Fourier transform, the Walsh transform is composed of even and odd Walsh functions. Any waveform f(t) can be written in terms of sums of the Walsh function series as follows:

$$f(t) = a_0 WAL(0,t) + \sum_{i=1}^{N/2-1} [a_i WAL(2i,t) + b_i WAL(2i+1,t)] \quad (19)$$

Signal Transforms

- The Discrete Walsh Transform

  In general WAL(i, t)=+1 or -1 and any two Walsh functions are orthogonal, that is,

$$\sum_{t=1}^{N} WAL(p,t)WAL(q,t) = \begin{cases} N & \text{for } p = q \\ 0 & \text{for } p \neq q \end{cases}$$

  The transform–inverse pair is written formally as follows:

$$X_k = \frac{1}{N}\sum_{i=1}^{N} x_i WAL(k,i) \tag{20}$$

  and the inverse is;

$$x_i = \sum_{i=1}^{N} X_k WAL(k,i) \tag{21}$$

- The Discrete Walsh Transform
  The discrete Walsh transform can be calculated using matrix multiplication;

$$X_K = x_i W_{ki} \tag{22}$$

where $\mathrm{x}_i = [\mathrm{x}_1, x_2, ..., x_N\ ]$ and W_{ki} is the N x N Walsh transform matrix.

Signal Transforms

- The Hadamard Transform
  - The Hadamard transform (also known as the Walsh–Hadamard transform,) is an example of a generalized class of Fourier transforms.
  - The Hadamard transform is similar to the Walsh transform but has rows of the transform matrix ordered differently.
  - The Hadamard transform is used in a number of applications, such as image processing, speech processing, filtering, and power spectrum analysis. It is very useful for reducing bandwidth storage requirements and spread-spectrum analysis.

  A Hadamard matrix H_{jj} is a symmetric JxJ matrix with elements $+1$ and -1. The Hadamard matrix of second order is given by

$$H_{22} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Signal Transforms

- The Hadamard Transform
  - A Hadamard matrix of order 2J can be written as

$$H_{2J2J} = \begin{pmatrix} H_{JJ} & H_{JJ} \\ H_{JJ} & -H_{JJ} \end{pmatrix}$$

-Inverse Hadamard matrices are easily computed as

$$H_{JJ}^{-1} = \frac{1}{J} H_{JJ}$$

- The Hadamard transform and its inverse are given by

$$F = H_{MM}.f.H_{NN}, f = \frac{1}{MN} H_{MM}.F.H_{NN},$$

-It can be seen that only matrix multiplication is necessary to compute a Hadamard transformation.

Signal Transforms

TABLE 3.1 Review of transforms and their demonstrations

Review of Transforms	Demonstration
Continuos-Time The Fourier Transform	$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$
Discrete-Time The Fourier Transform	$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$
The Wavelet Transform	$W(a,b) = \int_{-\infty}^{\infty} x(t)\, \overline{\Psi_{(a,b)}(t)}dt$
The z-Transform	$F(z) = \sum_{n=-\infty}^{\infty} f(n)z^{-n}$
Discrete Cosine Transform	$X_c(k) = \frac{1}{N}\sum_{n=1}^{N} x_n cos(\frac{\pi k(2n+1)}{2N})$
The Discrete Walsh Transform	$x_i = \sum_{i=1}^{N} X_k WAL(k,i)$ $X_K = x_i W_{ki}$
The Hadamard Transform	$F = H_{MM}.f.H_{NN}$

TABLE 3.2 Review of transforms and MATLAB Codes

REVIEW OF TRANSFORMS	MATLAB CODE	EXPLANATION
Fast Fourier transform	Y = fft(X)	computes the discrete Fourier transform (DFT) of X using a fast Fourier transform (FFT) algorithm.
The Wavelet Transform	wt = cwt(x)	returns the continuous wavelet transform (CWT) of x.
The Z-Transform	ztrans(f)	finds the Z-Transform of f.
Discrete Cosine Transform	y = dct(x)	returns the unitary discrete cosine transform of input array x. The output y has the same size as x.
Discrete Walsh-Hadamard Transform	y = fwht(new,N,'sequency')	perform Fast-walsh-hadamard-transform

Information in signals can be obtained from their power spectrum, where the most relevant measure is the autocorrelation function used to characterize the signal in the time domain.

AUTOCORRELATION and POWER DENSITY SPECTRUM

- A digital signal x(n) obtained by sampling some analog signal x(t).

The energy of an analog signal x(t) as

$$E = \int_{-\infty}^{\infty} (|x(t)|)^2 dt < \infty \tag{23}$$

If the signal has finite energy, then its Fourier transform exists

$$X(F) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi Ft} dt \tag{24}$$

Autocorrelation and Power Density Spectrum

By Parseval's theorem, we find that $(|X(n)|)^2$
is referred to as the energy density spectrum of the signal, Sxx. Here;

$$S_{xx} = (|X(n)|)^2 \tag{25}$$

In time domain, the Fourier transform of the energy density spectrum Sxx is called the autocorrelation function and written as

$$R_{xx}(\tau) = \int_{-\infty}^{\infty} x^*(t)x(t+\tau)dt \tag{26}$$

The power density spectrum is then the Fourier transform of this autocorrelation function written as;

$$\tau_{xx}(F) = \int_{-\infty}^{\infty} \gamma_{xx}(\tau)e^{-j2\pi Ft}dt \tag{27}$$

Nonparametric Estimation Models

The classical nonparametric methods that have been used for estimation of power density spectrum include the Bartlett, Welch, Blackman and Tukey methods.

- The Bartlett method reduces the variance observed in the periodogram by averaging the periodograms.
- The Welch computed a modified periodogram, which used just a selected part of the segment.
- In the Blackman and Tukey approach, problem is addressed by first windowing the sample autocorrelation sequence and then obtaining the power spectrum from the Fourier transform.

Parametric Estimation Models

The model used is based on the output of a linear system having the form.
1. Autoregressive (AR) Model
2. Moving Average Model
3. Autoregressive Moving Average Model

***The AR model is the most popular of the three.

Autoregressive (AR) Model

The AR model parameters can be estimated using several methods such as the Yule–Walker, Burg and covariance methods.

- The Yule– Walker method uses a biased form of the autocorrelation estimate.
- The Burg method estimates parameters by minimizing forward and backward errors of the linear system.
- The advantages of the Burg method are that it provides high-frequency resolution, gives a stable AR model, and is computationally efficient.

Moving Average Model - Autoregressive Moving Average Model

- Moving Average Model
  The noise whitening filter for the moving average process is regarded as an all-pole filter.
- Autoregressive Moving Average Model
  This model can improve on the AR model by using fewer model parameters and has been used in situations where the signal is corrupted by additive white noise.

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Thanks for listening...